

HW 17.3 # 1, 3, 5, 7, 11, 15 due 12/13/2020

Hari On

1.)  $\text{div } F = 0 + 0 + 0 = 0$

3.)  $\text{div } F = 2 + 0 + 0 = 2 \quad \int_0^2 \int_0^1 \int_0^1 2 \, dx \, dy \, dz = 4$

5.)  $\text{div } F = 0 + 0 + z^2 = z^2$

$$\int_0^{2\pi} \int_0^{\pi} \int_0^1 r^3 \cos \theta \sin \theta \, dr \, d\theta \, d\phi = \int_0^{2\pi} \int_0^{\pi} \frac{1}{4} \cos \theta \sin \theta \, d\theta \, d\phi = \int_0^{2\pi} \int_0^{\pi} \frac{1}{8} \sin 2\theta \, d\theta \, d\phi$$

$$= \int_0^{2\pi} \left. -\frac{1}{16} \cos 2\theta \right|_0^{\pi} d\phi = \int_0^{2\pi} 0 \, d\phi = 0$$

7.)  $\text{div } F = y^2 + z^2 + x^2 = r^2 + z^2$

$$\int_0^{2\pi} \int_0^3 \int_0^2 r^3 + rz^2 \, dz \, dr \, d\theta = \int_0^{2\pi} \int_0^3 \left. zr^3 + \frac{rz^3}{3} \right|_0^2 dr \, d\theta = \int_0^{2\pi} \int_0^3 2r^3 + \frac{8}{3}r \, dr \, d\theta$$

$$= \int_0^{2\pi} \left. \frac{r^4}{2} + \frac{4}{3}r^2 \right|_0^3 d\theta = \int_0^{2\pi} \frac{81}{2} + \frac{24}{2} \, d\theta = \int_0^{2\pi} \frac{105}{2} \, d\theta = 105\pi$$

11.)  $\text{div } F = 3x^2 + 3z^2 = 3(r^2 \sin^2 \theta \cos^2 \theta + r^2 \cos^2 \theta)$

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^2 3r^3 \sin^2 \theta \cos^2 \theta \sin \theta + 3r^3 \cos^2 \theta \sin \theta \, dr \, d\theta \, d\phi$$

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} 12 \sin^2 \theta \cos^2 \theta \sin \theta + 12 \cos^2 \theta \sin \theta \, d\theta \, d\phi = \int_0^{\frac{\pi}{2}} \left. -4 \sin^2 \theta \cos^3 \theta - 12 \cos^2 \theta \cos \theta \right|_0^{\frac{\pi}{2}} d\theta$$

$$\int_0^{\frac{\pi}{2}} 4 \sin^2 \theta + 12 \cos^2 \theta \, d\theta = 8\theta + 2 \sin(2\theta) \Big|_0^{\frac{\pi}{2}} = 4\pi$$

15.)  $\text{div } F = 1 + 0 + 1 = 2$

$$\int_{-3}^3 \int_{-3}^3 \int_0^{9-x^2-y^2} 2 \, dz \, dy \, dx = \int_{-3}^3 \int_{-3}^3 18 - 2x^2 - 2y^2 \, dy \, dx = \int_{-3}^3 \left. 18y - 2x^2y - \frac{2}{3}y^3 \right|_{-3}^3 dx$$

$$\int_{-3}^3 (54 - 6x^2 - 18) - (-54 + 6x^2 + 18) \, dx = \int_{-3}^3 72 - 12x^2 \, dx = 72x - 4x^3 \Big|_{-3}^3 = (216 - 108) - (-216 + 108)$$

$$= 216$$