17.3

1. $F(x,y,z) = \langle z, x, y \rangle$.

$\operatorname{div} F = \frac{d}{dx} z + \frac{d}{dy} x + \frac{d}{dz} y$

$= 0+0+0$

$= 0$

3. $F(x,y,z) = \langle 2x, 3z, 3y \rangle$

$\operatorname{div} F = \frac{d}{dx} 2x + \frac{d}{dy} 3z + \frac{d}{dz} 3y$

$= 2 + 0 + 0 = 2$

$0 \le r \le 1$ $\quad 0 \le \theta \le 2\pi$ $\quad 0 \le z \le 2$

$\iiint \operatorname{div} F \, dV$

$= \int_0^{2\pi}\int_0^1\int_0^2 2 \, dz \, r \, dr \, d\theta$

$= 4\pi$

5. $F(x,y,z) = \langle 0, 0, \frac{z^3}{3} \rangle$

$\operatorname{div} F = \frac{d}{dx} 0 + \frac{d}{dy} 0 + \frac{d}{dz} \frac{z^3}{3}$

$= z^2$

~~there~~ I don't know how to get the region of z.

7. $F(x,y,z) = \langle xy^2, yz^2, zx^2 \rangle$

$\operatorname{div} F = \frac{d}{dx} xy^2 + \frac{d}{dy} yz^2 + \frac{d}{dz} zx^2$

$= y^2 + z^2 + x^2 = r^2 + z^2$

$x = r\cos\theta$ $\quad y = r\sin\theta$

$0 \le r \le 2$ $\quad 0 \le \theta \le 2\pi$ $\quad 0 \le z \le 3$

$\int_0^{2\pi}\int_0^2\int_0^3 (r^2 + z^2) \, r \, dz \, dr \, d\theta$

$= 60\pi$

11. $F(x,y,z) = \langle x^3, 0, z^3 \rangle$

$\operatorname{div} F = \frac{d}{dx} x^3 + \frac{d}{dy} 0 + \frac{d}{dz} z^3$

$= 3x^2 + 0 + 3z^2 = 3x^2 + 3z^2$

$= 3(x^2 + z^2)$

$x = \rho\cos\theta\sin\phi$ $\quad y = \rho\sin\theta\sin\phi$ $\quad z = \rho\cos\phi$

$dV = \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$0 \le \rho \le 2$ $\quad 0 \le \theta \le \cancel{2\pi} \frac{\pi}{2}$ $\quad 0 \le \phi \le \frac{\pi}{2}$

$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^2 3\left((\rho\cos\theta\sin\phi)^2 + (\rho\cos\phi)^2\right) \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta = \frac{32}{5}\pi$

15. $F(x,y,z) = \langle x+y, z, z-x \rangle$

$$\operatorname{div} F = \frac{d}{dx}\, x+y + \frac{d}{dy}\, z + \frac{d}{dz}\, z-x$$

$$= 1 + 0 + 1 = 2$$

$z=0 \quad 0 = 9 - x^2 - y^2$

$x^2 + y^2 = 9$

$0 \le r \le 3 \quad 0 \le \theta \le 2\pi \qquad 0 \le z \le 9 - x^2 - y^2$

$$\cancel{\int_0^{2\pi}\int_0^3\int_0^{9-x^2-y^2}}$$

$$\int_0^{9-x^2-y^2} 2\, dz$$

$$= 2z \Big|_0^{9-x^2-y^2}$$

$$= 2(9-x^2-y^2) = 18 - 2x^2 - 2y^2$$

$$= 18 - 2(x^2+y^2)$$

$$= \cancel{18-2r}\ 18 - 2r^2$$

$$\int_0^{2\pi}\int_0^3 (18 - 2r^2)\, r\, dr\, d\theta$$

$$= 81\pi$$