Calc HW Due 12/13

Section 17.3
1, 3, 5, 7, 11, 15

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① $F(x,y,z) = \langle z, x, y \rangle$ the box $[0,4] \times [0,2] \times [0,3]$

ⓘ Find the divergence of F

$$\text{div } F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$= 0 + 0 + 0 = 0$$

$$\iint_S F \cdot dS = \iiint_E \text{div } F \, dV$$

$$= \int_0^4 \int_0^2 \int_0^3 0 \, dz\, dy\, dx = \boxed{0}$$

③ $F(x,y,z) = \langle \underset{P}{2x}, \underset{Q}{3z}, \underset{R}{3y} \rangle$, the region $x^2+y^2 \leq 1, 0 \leq z \leq 2$

$$\text{div} F = 2 + 0 + 0 = 2$$

$$\int_0^2 \int_0^1 \int_0^{\sqrt{1-y^2}} 2 \, dx\, dy\, dz = \int_0^2 \int_0^{2\pi} \int_0^1 2 \, r dr\, d\theta\, dz$$

Convert To Spherical Coordinates (Polar)

$$\int_0^2 \int_0^{2\pi} \int_0^1 2 \rho^2 \sin\theta \, d\rho\, d\theta\, d\phi = \boxed{4\pi}$$

⑤ Evaluate $F(x,y,z) = \langle 0, 0, z^3/3 \rangle$, S is the sphere $x^2+y^2+z^2=1$

$$\text{div}(F) = 0 + 0 + z^2 = z^2$$

$$\iiint_W z^2 \, dV = \int_0^{2\pi} \int_0^{\pi} \int_0^1 \rho^2 \cos^2\phi \, \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

Outer: $\int_0^{2\pi} d\theta = 2\pi$ · Middle: $\int_0^{\pi} \cos^2\phi \sin\phi \, d\phi = \left. \frac{-\cos^3\phi}{3} \right]_0^{\pi} = \frac{2}{3}$ · Outer: $\int_0^1 \rho^4 \, d\rho = \frac{1}{5}$ $= \boxed{\frac{4\pi}{15}}$

⑦ $F(x,y,z) = \langle xy^2, yz^2, zx^2 \rangle$, S is the boundary of the cylinder given by $x^2+y^2 \le 4, 0 \le z \le 3$

$div(F) = y^2 + z^2 + x^2$ convert to polar

$$\int_0^3 \int_0^{2\pi} \int_0^2 ((r\sin\theta)^2 + z^2 + (r\cos\theta)^2)\, r\,dr\,d\theta\,dz$$

$r^2\sin^2\theta + z^2 + r^2\cos^2\theta$

$$= \int_0^3 \int_0^{2\pi} \int_0^2 (r^2 + z^2)\, r\,dr\,d\theta\,dz = \boxed{60\pi}$$

⑪ $F(x,y,z) = \langle x^3, 0, z^3 \rangle$, S is boundary of the region in the first octant of space given by $z^2+x^2+y^2 \le 4$, $x \ge 0, y \ge 0, z \ge 0$

$div(F) = 3x^2 + 0 + 3z^2 = 3(x^2 + z^2)$

Done On Maple

$$3\int_0^{\pi/2}\int_0^{\pi/2}\int_0^2 (\rho\cos\theta\sin\phi)^2 + (\rho\cos\phi)^2\, \rho^2\sin\theta\, d\rho\, d\phi\, d\theta$$

$$= \frac{32\pi}{5}$$

⑮ $F(x,y,z) = \langle x+y, z, z-x \rangle$, S is the Boundary of the region between the paraboloid $z = 9 - x^2 - y^2$ and the xy plane

$9 - r^2$

$div(F) = 1 + 0 + 1 = 2$

Convert To Polar

r

$$\int_0^3 \int_0^{2\pi} \int_0^{9-r^2} 2\, dz\, d\theta\, r\,dr$$

$$= 2\int_0^3 \int_0^{2\pi} \int_0^{9-r^2} r\, dz\, d\theta\, dr$$

Inner: $9r - r^3$

Middle: $\int_0^{2\pi} 9r - r^3\, d\theta = 2\pi(9r - r^3) = 18\pi r - 2\pi r^3$

Outer: $2\int_0^3 18\pi r - 2\pi r^3\, dr = 18\pi \frac{r^2}{2}\Big]_0^3 - \pi\left[\frac{r^4}{4}\right]_0^3 \Rightarrow 162\pi - 81\pi = \boxed{81\pi}$

$2(81\pi - \frac{81\pi}{2})$