1 3 5 7 11 15 17.3 HW Orion Kress-Santilippo

1) $F(x,y,z) = \langle z, x, y \rangle$ box $[0,4] \times [0,2] \times [0,3]$

$$div(F) = 0 + 0 + 0 = 0$$

$$Ans = \int_0^3 \int_0^2 \int_0^4 (div(F))\, dx\, dy\, dz = \int_0^3 \int_0^2 \int_0^4 0\, dx\, dy\, dz$$

$$= \boxed{0}$$

3) $F = \langle 2x, 3z, 3y \rangle$

$$div(F) = 2 + 0 + 0 = 2$$

$x^2 + y^2 \le 1$ $\quad x = r\cos\theta$, $y = r\sin\theta$

$0 \le z \le 2$

Cylinder w/ Volume $\pi(1)^2 \cdot 2 = 2\pi$

$$= \int_0^2 \int_0^{2\pi} \int_0^1 2r\, dr\, d\theta\, dz$$

$$= \text{Volume} \cdot \text{Integrand} = \boxed{4\pi}$$

17.3 Cont

5) $F = \langle 0, 0, \frac{z^3}{3} \rangle$ S: Sphere rad 1 cent. @ origin

$\operatorname{div}(F) = z^2$ $\quad z = \rho\cos\phi$

$$\iiint z^2\,dx\,dy\,dz = \int_0^{\pi}\int_0^{2\pi}\int_0^{1} \rho^3\cos^2\phi\sin\phi\,d\rho\,d\theta\,d\phi$$

$$= \frac{4\pi}{15}$$

7) $F = \langle xy^2, yz^2, zx^2 \rangle$ S: Cylinder rad. 2 height 3 cent. @ origin

$\operatorname{div}(F) = y^2 + z^2 + x^2$ $\quad y^2 + x^2 = r^2$ $\quad 0 \le r \le 2$, $0 \le z \le 3$, $0 \le \theta \le 2\pi$

• Ans

$$= \int_0^3\int_0^{2\pi}\int_0^2 (r^2 + z^2)\,r\,dr\,d\theta\,dz$$

$$= 60\pi$$

17.3 Cont

11) $F = \langle x^3, 0, z^3 \rangle$ S: First octant of sphere rad 2 cent @ origin

$div(F) = 3x^2 + 3z^2$

$x = \rho \sin\phi \cos\theta$

$z = \rho \cos\phi$

$$\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_0^{2} 3(\rho \sin\phi \cos\theta)^2 + 3(\rho \cos\phi)^2 \, d\rho \, d\theta \, d\phi$$

$$= \frac{32\pi}{5}$$

15) $F = \langle x+y, z, z-x \rangle$

$div(F) = 1 + 0 + 1 = 2$

$$Ans = \int_0^3 \int_0^3 \int_0^{9-x^2-y^2} 2 \, dz \, dx \, dy = 81\pi$$