17.3 : 1, 3, 5, 7, 11, 15

1) $F(x,y,z) = \langle z, x, y \rangle$, the box $[0,4] \times [0,2] \times [0,3]$

$\iint_S F\,dS = \iiint_R \operatorname{div}(F)\,dV \qquad \iiint 0 = 0$

$\operatorname{div}(F) = 0 + 0 + 0 = 0$

3) $F(x,y,z) = \langle 2x, 3z, 3y \rangle$, the region $x^2 + y^2 \le 1$, $0 \le z \le 2$

$\operatorname{div}(F) = 2 + 0 + 0 = 2$

$D: \{(r,\theta,z) \mid 0 \le r \le 1,\ 0 \le \theta \le \pi,\ 0 \le z \le 2\}$

$\int_0^1 \int_0^{\pi} \int_0^2 2\,dz\,d\theta\,dr \rightarrow 2z\Big|_0^2 = 4$

$\int_0^{2\pi} 4\,d\theta = 4\theta\Big|_0^{\pi} = 4\pi$

5) $F(x,y,z) = \langle 0, 0, z^3/3 \rangle$, S is the sphere $x^2 + y^2 + z^2 = 1$

$\operatorname{div}(F) = 0 + 0 + z^2 = z^2$

$D: \{(r,\theta,z) \mid 0 \le r \le 1,\ 0 \le \theta \le 2\pi,\ -1 \le z \le 0\}$

$\int_0^1 \int_0^{2\pi} \int_{-1}^0 z^2\,dz\,d\theta\,dr \rightarrow 2z\Big|_{-1}^0 = 2$

$\int_0^{2\pi} \frac{2}{15}\,d\theta = \frac{2\theta}{15} = \frac{4\pi}{15}$

7) $F(x,y,z) = \langle xy^2, yz^2, zx^2 \rangle$, S the boundary of the cylinder given by $x^2 + y^2 \le 4$, $0 \le z \le 3$

$\operatorname{div}(F) = y^2 + z^2 + x^2$

$D: \{(r,\theta,z) \mid 0 \le r \le 2,\ 0 \le \theta \le 2\pi,\ 0 \le z \le 3\}$

$\int_0^2 \int_0^{2\pi} \int_0^3 \sin^2\theta + z^2 + \cos^2\theta\,dz\,d\theta\,dr$

$z\sin^2\theta + \frac{z^3}{3} + z\cos^2\theta\Big|_0^3 = 3(\sin^2\theta + \cos^2\theta + 3)$
$= 15$

$\int_0^{2\pi} 15\,d\theta = 15\theta\Big|_0^{2\pi} = 30\pi$

$\int_0^2 30\pi\,dr = 30\pi r\Big|_0^2 = 60\pi$

11) $F(x,y,z) = \langle x^3, 0, z^3 \rangle$, S is boundary of the region in the first octant of s ace given by $x^2 + y^2 + z^2 \le 4$, and $x \ge 0, y \ge 0, z \ge 0$

$\operatorname{div}(F) = 3x^2 + 0 + 3z^2$

$D: \{(r,\theta,z) \mid 0 \le r \le 2,\ 0 \le \theta \le \frac{\pi}{2},\ 0 \le z \le 2 - r^2\}$

$\int_0^2 \int_0^{\frac{\pi}{2}} \int_0^{2-r^2} 3\cos^2\theta + 3z^2\,dz\,d\theta\,dr$

$3z\cos^2\theta + z^3\Big|_0^{2-r^2} \quad (6 - 3r^2)\cos^2\theta + 2 - r^2$

$\int_0^{\frac{\pi}{2}} (6 - 3r^2)\cos^2\theta + 2 - r^2\,d\theta$

$= (6 - 3r^2)\left(\frac{1}{2}\left(\theta + \frac{1}{2}\sin(2\theta)\right)\right) + 2\theta - \theta r^2\Big|_0^{\frac{\pi}{2}}$

$= (6 - 3r^2)\left(\frac{1}{2}\left(\frac{\pi}{2} + \cancel{\tfrac{1}{2}(\sin\pi)}^{\,0}\right)\right) + \pi - \frac{\pi}{2}r^2$

$\int_0^2 \frac{\pi}{4}(6 - 3r^2) + \pi - \frac{\pi}{3}r^2\,dr$

$\frac{\pi}{4}(6 - r^3) + \pi - \frac{\pi}{9}r^3\Big|_0^2$

$= \frac{\pi}{4}(6 - 8) + \pi - \frac{\pi}{9}(8) = \frac{32\pi}{5}$

15) $F(x,y,z) = \langle x + y, z, z - x \rangle$, S is the boundary of the region between the paraboloid $z = 9 - x^2 - y^2$ and the xy-plane

$\operatorname{div}(F) = 1 + 0 + 1 = 2$

$D: \{(r,\theta,z) \mid 0 \le r \le 3,\ 0 \le \theta \le 2\pi,\ 0 \le z \le 9 - r^2\}$

$\int_0^3 \int_0^{2\pi} \int_0^{9-r^2} 2\,dz\,d\theta\,dr$

$2z\Big|_0^{9-r^2} = 18 - 2r^2$

$\int_0^{2\pi} 18 - 2r^2\,d\theta$

$18\theta - 2r^2\theta\Big|_0^{2\pi}$

$= 39\pi - 4\pi r^2$

$\int_0^3 39\pi - 4\pi r^2\,dr$

$39\pi r - \frac{4}{3}\pi r^3\Big|_0^3$

$117\pi - 36\pi = 81\pi$