1. $F(x,y,z) = \langle z, x, y \rangle$ the box $[0,4] \times [0,2] \times [0,3]$

$$\operatorname{div} F = \frac{d}{dx}(z) + \frac{d}{dy}(x) + \frac{d}{dz}(y)$$

$$= 0$$

$$\iiint_E \operatorname{div} F \cdot dV$$

$$= \int_0^3 \int_0^2 \int_0^4 0 \, dx\,dy\,dz$$

$$= 0$$

3. $F(x,y,z) = \langle 2x, 3z, 3y \rangle$ $x^2+y^2 \le 1,\ 0 \le z \le 2$

$$\operatorname{div} F = \frac{d}{dx}(2x) + \frac{d}{dy}(3z) + \frac{d}{dz}(3y)$$

$$= 2$$

$0 \le \theta \le 2\pi,\ 0 \le r \le 1,\ 0 \le z \le 2$

$$\iint_S F \cdot ds = \iiint_R 2r \cdot dz\,dr\,d\theta$$

$$= \int_0^{2\pi} \int_0^1 \int_0^2 2r\,dz\,dr\,d\theta$$

$$= 4\pi$$

5. $F(x,y,z) = \langle 0, 0, \frac{z^3}{3} \rangle$, $x^2+y^2+z^2 = 1$

$$\operatorname{div} F = \frac{d}{dz}\left(\frac{z^3}{3}\right) = z^2$$

$x = \cos u \cos v\, r$, $y = \cos u \sin v\, r$, $z = \sin u\, r$

$$\begin{vmatrix} x_r & y_r & z_r \\ x_u & y_u & z_u \\ x_v & y_v & z_v \end{vmatrix} = -r^2 \cos u$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\pi} \int_0^1 -r^2 \cos u \cdot r^2 \sin^2 u \, dr\,du\,dv$$

$$= \frac{4\pi}{15}$$

7. $F(x,y,z) = \langle xy^2, yz^2, zx^2 \rangle$, $x^2+y^2 \le 4$, $0 \le z \le 3$

$\text{div} F = \frac{d}{dx}(xy^2) + \frac{d}{dy}(yz^2) + \frac{d}{dz}(zx^2) = x^2+y^2+z^2$

$$\iint_{x^2+y^2\le 4} \int_0^3 x^2+y^2+z^2 \, dz \, dy \, dx$$

$$= 3 \iint_{x^2+y^2\le 4} (x^2+y^2+3) \, dy \, dx$$

$x = r\cos\theta$, $y = r\sin\theta$

$0 \le \theta \le 2\pi$, $0 \le r \le 2$

$$3 \int_0^{2\pi} \int_0^{r} (r^2+3) r \, dr \, d\theta$$

$= 60\pi$

11. $F(x,y,z) = \langle x^3, 0, z^3 \rangle$

$\text{div} F = 3x^2 + 3z^2$

$x = r\sin u \cos v$, $y = r \sin u \sin v$, $z = r\cos u$

$0 \le r \le 2$, $0 \le u \le 2\pi$, $0 \le v \le 2\pi$

$$\int_0^{2\pi} \int_0^{2\pi} \int_0^{2} (3(r^2 \sin^2 u \cos^2 v + r^2 \cos^2 u) r^2 \sin u) \, dr \, du \, dv$$

$= \frac{32\pi}{5}$

15. $F(x,y,z) = \langle x+y, z, z-x \rangle$ $z = 9 - x^2 y^2$