17.3 1, 3, 5, 7, 11, 15

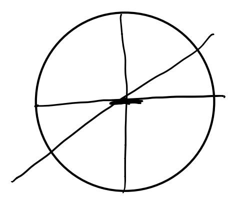
#1 $F(x,y,z) = \langle z, x, y \rangle$, the box $[0,4] \times [0,2] \times [0,3]$

div $F = 0+0+0 = 0$

$\int 0 \, dV = \boxed{0}$ No need to find the actual volume because div = 0

#3 $F(x,y,z) = \langle 2x, 3z, 3y \rangle$ the region $x^2+y^2 \leq 1, 0 \leq z \leq 2$

div $F = 2$



$A = 4\pi r^2$

$A = 4\pi$

$$\int_0^{\pi} \int_0^1 \int_0^2 2 \cdot 2\pi \cdot r \, dz\,dr\,d\theta$$

$$\int_0^2 4\pi r \, dz = 4\pi r z \Big|_0^2$$

$$\int_0^1 8\pi r \, dr = 4\pi r^2 \Big|_0^1$$

$$\int_0^{\pi} 4\pi \, d\theta = \boxed{4\pi^2}$$

#5 $F(x,y,z) = \langle 0, 0, z^3/3 \rangle$, S is the sphere $x^2+y^2+z^2=1$

div $F = z^2$ $4\pi r^2$

$$\int_0^{\pi} \int_0^{2\pi} \int_0^1 r^4 \cos^2\phi \sin\phi \, dr\,d\theta\,d\phi$$

$$= \int_0^1 r^4 \cos^2\phi \sin\phi \, dr = \cos^2\phi \sin\phi \frac{r^5}{5} \Big|_0^1$$

$$\frac{1}{5} \int_0^{2\pi} \cos^2\phi \sin\phi \, d\theta = \frac{1}{5} \cdot \theta \cos^2\phi \sin\phi \Big|_0^{2\pi}$$

$$\frac{2\pi}{5} \int_0^{\pi} \cos^2\phi \sin\phi \, d\phi$$ $u = \cos\phi$ $du = -\sin\phi \, d\phi$

$$-\frac{2\pi}{5} \int_{-1}^{1} u^2 \, du \quad \frac{u^3}{3} \Big|_{-1}^{1} = \frac{2\pi}{5} \cdot \left(\frac{1}{3} - -\frac{1}{3}\right) = \boxed{\frac{4\pi}{5}}$$

7 $F = \langle x,y,z \rangle = \langle xy^2, yz^2, zx^2 \rangle$, S is the boundary of the cylinder given by $x^2+y^2 \le 4$, $0 \le z \le 3$

$$\text{div } F = y^2 + z^2 + x^2$$

$$\int_0^{2\pi}\int_0^{2}\int_0^{3} r^2 + z^2 \, dz\, dr\, d\theta$$

$$\int_0^3 r^2 + z^2 \, dz = zr^2 + \frac{z^3}{3}\Big|_0^3$$

$$\int_0^2 3r^2 + 9 \, dr = r^3 + 9r \Big|_0^2$$

$$\int_0^{2\pi} 26 \, d\theta = 26\theta \Big|_0^{2\pi} = \boxed{52\pi}$$

11 $F(x,y,z) = \langle x^3, 0, z^3 \rangle$, S is the boundary of the region in the first octant of space given by $x^2+y^2+z^2 \le 4$ and $x \ge 0$, $y \ge 0$, $z \ge 0$

$$\text{div } F: 3x^2 + 3z^2$$

$$3\int_0^{\pi/2}\int_0^{2\pi}\int_0^{2} (p^2\cos^2\theta\sin^2\phi + p^2\cos^2\phi) \cdot p^2 \sin\phi \, dp\, d\theta\, d\phi$$

$$3\int_0^{2} (p^2\cos^2\theta\sin^2\phi + p^2\cos^2\phi) \cdot p^2 \sin\phi \, dp$$

$$= \frac{p^5}{5}(\cos^2\theta\sin^2\phi + \cos^2\phi)\sin\phi \Big|_0^2 =$$

$$3\int_0^{2\pi} \frac{32}{5} \cos^2\theta \sin^3\phi + \cos^2\phi\sin\phi \, d\theta = \frac{96\pi}{5} \frac{\sin^3\theta}{3} \cdot \frac{\sin^3\phi}{1} + \cos^2\phi\sin\phi \Big|$$

$$= \frac{96\pi}{5}\int_0^{\pi/2} \cos^2\phi \sin\phi \, d\phi = \int_0^1 u^2 \, du = \frac{u^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$\frac{96\pi}{5} \cdot \frac{1}{3} = \boxed{\frac{32\pi}{5}}$$

#15 $F(x,y,z) = \langle x+y, z, z-x \rangle$, S is the boundary of the region between the paraboloid $z = 9 - x^2 - y^2$ and the x-y plane.

$\operatorname{div} F: \ 1 + 0 + 1 = 2$

$$\int_0^{2\pi} \int_0^3 2 \cdot (9 - x^2 - y^2) \, dr \, d\theta$$

$$2 \int_0^{2\pi} \int_0^3 (9 - r^2) \, r \, dr \, d\theta$$

$$2 \int_0^{2\pi} \int_0^3 9r - r^3 \, dr \, d\theta$$

$$\int_0^3 9r - r^3 \, dr = 9\frac{r^2}{2} - \frac{r^4}{4} \Big|_0^3$$

$$2 \int_0^{2\pi} \frac{81}{2} - \frac{81}{4} \, d\theta = \frac{81}{2} \theta \Big|_0^{2\pi} = 81\pi - 0 = \boxed{81\pi}$$