

17.3

1. $\text{div}(F) = 0$

$$\iiint 0 \, dV = 0$$

3. $\text{div}(F) = 2 + 0 + 0$

$$\iiint 2 \, dV$$

$$\pi(2^3) = 4\pi$$

$$\frac{4\pi}{2} = 2\pi \quad 2\pi \cdot \iiint 2 \, dV = 4\pi$$

5. $\text{div}(F) = 0 + 0 + z^2$

$$x \leq 1 \quad y \leq 1 \quad z \leq 1$$

$$\iiint z^2 \, dV$$

$$\int_0^1 \int_0^1 \int_0^1 p^2 \cos^2 \phi \cdot p^2 \sin \phi \, dp \, d\phi \, d\theta$$

$$\int_0^1 \int_0^{2\pi} \int_0^1 \frac{p^5}{5} \cdot \cos^2 \phi \cdot \sin \phi \, d\phi \, d\theta$$

$$\frac{1}{5} \int_0^{2\pi} \int_0^{\pi} \frac{1}{3} \cos^3 \phi \, d\phi \, d\theta$$

$$\frac{2}{15} \int_0^{2\pi} 1 \, d\theta$$

$$\frac{4\pi}{15}$$

$$7. \operatorname{div}(F) = y^2 + z^2 + x^2 \quad 0 \leq \theta \leq 2\pi \quad 0 \leq r \leq 2 \quad 0 \leq z \leq 3$$

$$\int_0^{2\pi} \int_0^2 \int_0^3 (z^2 + r^2) r \, dz \, dr \, d\theta$$

$$\int_0^{2\pi} \int_0^2 \left(\frac{z^3}{3} \cdot r + z r^3 \right) \Big|_0^3 \, dr \, d\theta$$

$$\int_0^{2\pi} \left(\frac{9r^2}{2} + \frac{3r^4}{4} \right) \Big|_0^2 \, d\theta$$

$$\int_0^{2\pi} 30 \, d\theta$$

$$60\pi$$

$$11. \operatorname{div}(F) = 3x^2 + 3z^2$$

$$3 \int_0^{\pi/2} \int_0^{\pi/2} \int_0^2 p^2 \sin^2 \phi \cos^2 \theta + p^2 \cos^2 \phi \sin^2 \theta \, dp \, d\phi \, d\theta$$

$$3 \int_0^{\pi/2} \int_0^{\pi/2} \left(\frac{p^3}{3} \right) \Big|_0^2 (\sin^2 \phi \cos^2 \theta + \sin^2 \theta \cos^2 \phi) \, d\phi \, d\theta$$

$$\frac{96}{5} \int_0^{\pi/2} \int_0^{\pi/2} \sin^4 \phi \cdot \frac{1}{4 \cos^2 \theta} - \cos^2 \theta \cos^2 \phi \, d\phi \, d\theta$$

$$\frac{96}{5} \int_0^{\pi/2} (0 \cos^2 \theta - 0 \cos^2 \theta) - (0 - 1 \cos^2 \theta) \, d\theta$$

$$\frac{96}{5} \left(\frac{\pi}{2} - 0 \right) = \frac{32\pi}{5}$$

$$15. \operatorname{div}(F) = 1 + 0 + 1$$

$$\iiint 2 \, dV$$

Volume of paraboloid

$$9^2 \cdot \frac{1}{2} \cdot \iiint 2 \, dV = 81\pi$$