homework 13

17.3

1.) $F(x,y,z) = \langle z, x, y \rangle$

divergence of F: 0

3.) $F(x,y,z) = \langle 2x, 3z, 7y \rangle$

$x^2+y^2 \le 1$, $0 \le z \le 2$

div F = 2

$$\iiint_R 2\,dV \to \int_0^{2\pi}\int_0^1\int_0^2 r\,dz\,dr\,d\theta \to \int_0^{2\pi}\int_0^1 r z\Big|_0^2\,dr\,d\theta$$

$$\int_0^{2\pi} 2\left|\frac{r^2}{2}\right|_0^1 d\theta \to \theta\Big|_0^{2\pi} = 2\pi \cdot 2 = \boxed{4\pi}$$

5.) $F = \langle 0, 0, \frac{z^3}{3} \rangle$

div F: z^2

$0 \to 1$, $0 \to \pi$, $0 \to 2\pi$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^1 z^2 \rho \sin\phi\,d\rho\,d\phi\,d\theta \to \int_0^{2\pi}\int_0^{\pi}\int_0^1 \rho\cos\phi\,\rho\sin\phi\,d\rho\,d\phi\,d\theta$$

$$\int_0^{2\pi} d\theta \cdot \int_0^{\pi} \cos\phi\sin\phi \int_0^1 \rho^2\,d\rho = 2\pi \cdot \frac{2}{5} \cdot \frac{1}{3} = \boxed{\frac{4}{15}\pi}$$

7.) $F = \langle xy^2, yz^2, 8x^2 \rangle$

div $F = y^2 + z^2 + x^2$

$$\iiint (r^2\cos^2\theta + r^2\sin^2\theta + z^2)\,r\,dz\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^2\int_0^3 (r^3 + r z^2)\,dz\,dr\,d\theta = \int_0^{2\pi}\int_0^2 3r^3 + 9r\,dr\,d\theta$$

$$3\int_0^{2\pi} \frac{16}{4} + \frac{12}{2} = 20\int_0^{2\pi} d\theta = \boxed{40\pi}$$

11.) $F = \langle x^3, 0, z^3 \rangle$

div $F = 3$

$3(\sin^2\theta\cos^2\varphi + \cos^2\theta)\rho^2$

$$3\int_0^{\pi/2}\int_0^{\pi/2}\int_0^2 \rho^4(\sin^3\theta\cos^2\varphi + \sin\theta\cos\theta)\, d\rho\, d\varphi\, d\theta$$

$$\frac{96}{5}\int_0^{\pi/2}\int_0^{\pi/2} \sin^3\theta\cos^2\varphi + \sin\theta\cos\theta\, d\varphi\, d\theta$$

$$\frac{96}{5}\int_0^{\pi/2} \cos^2\theta\left(\frac{2}{3}\right) + \frac{1}{3}\, d\theta = \frac{2}{3}\cdot\frac{96}{5}\cdot\frac{\pi}{4} + \frac{\pi}{5} = \boxed{\frac{32\pi}{5}}$$

15.) $F(x,y,z) = \langle x+y, z, z-x \rangle$

$z = 9 - x^2 - y^2$

div $F = 2$

$$2\int_0^{2\pi}\int_0^3\int_0^{9-r^2} r\, dz\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^3 r(9-r^2)\, dr\, d\theta \rightarrow 2\int_0^{2\pi}\frac{81}{4} = 2\cdot 2\pi\cdot\frac{81}{4} = \boxed{81\pi}$$