HW 17.3 due 12/13/20

1. $F(x,y,z) = \langle z, x, y\rangle$ the box $[0,4]\times[0,2]\times[0,3]$

$\iint_S F\cdot dS = \iiint_R \text{div}(F)\,dV$

$\text{div}(F) = 0+0+0 = 0 \qquad \iiint_R 0\,dV = \boxed{0}$

3. $F(x,y,z) = \langle 2x, 3z, 3y\rangle \qquad x^2+y^2 \le 1,\ 0\le z\le 2$

$\iint_S F\cdot dS = \iiint_R \text{div}(F)\,dV$

$\text{div}(F) = 2 \rightarrow 2\int_0^{2\pi}\int_0^1\int_0^2 r\,dz\,dr\,d\theta = \boxed{4\pi}$

5. $F(x,y,z) = \langle 0, 0, z^3/3\rangle \qquad x^2+y^2+z^2 = 1$

$\iint_S F\cdot dS \Rightarrow \iiint_R \text{div}(F)\,dV$

$\text{div}(F) = z^2 = \rho^2\cos^2\phi \qquad \iiint \rho^4\cos^2\phi\sin\phi\,d\rho\,d\phi\,d\theta$

$\int_0^{2\pi}\int_0^{2\pi}\int_0^1 \rho^4\cos^2\phi\sin\phi\,d\rho\,d\phi\,d\theta = \boxed{4\pi/15}$

7. $F(x,y,z) = \langle xy^2, yz^2, zx^2\rangle \qquad \text{div}F = y^2+z^2+x^2$

$\text{div}F = x^2+y^2+z^2 = r^2+z^2 \qquad \int_0^{2\pi}\int_0^2\int_0^3 r^3+rz^2\,dz\,dr\,d\theta = \boxed{160\pi}$

11. $F(x,y,z) = \langle x^3, 0, z^3\rangle \qquad x^2+y^2+z^2 \le 4 \quad x\ge 0, y\ge 0, z\ge 0$

$\text{div}\,F = 3x^2+3z^2 \qquad 3\iiint x^2+z^2\,dV \rightarrow$ Convert to Spherical

$x = \rho\sin\phi\cos\theta \qquad z = \rho\cos\phi$

$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^2 [(\rho\sin\phi\cos\theta)^2 + (\rho\cos\phi)^2]\,\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

$= \boxed{32\pi/5}$

15. $F(x,y,z) = \langle x+y, z, z-x\rangle \qquad z = 9-x^2-y^2 \quad$ xy-plane

$\text{div}F = 1+1 = 2 \rightarrow 2\iiint dV \qquad z = 9-r^2$

$2\int_0^{2\pi}\int_0^3\int_0^{9-r^2} dz\,dr\,d\theta \rightarrow \boxed{81\pi}$

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