17.3 HW - #1, 3, 5, 7, 11, 15

1. $F(x,y,z) = \langle z, x, y \rangle$

$\text{div}\vec{F} = \frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz}$

$\text{div}F = 0 + 0 + 0 = 0$

$\iiint_R 0\, dV = 0$

3. $F(x,y,z) = \langle 2x, 3z, 3y \rangle$

$x^2 + y^2 \leq 1$

$0 \leq z \leq 2$

$\text{div}\vec{F} = 2 + 0 + 0 = 2$

$\int_0^{2\pi}\int_0^1 \int_0^2 (2)\, dz\, dr\, d\theta$

$= \int_0^1 4r\, dr = 2$

$= \int_0^{2\pi} 2\, d\theta = 4\pi$

5. $\iint_S F \cdot dS$

$F(x,y,z) = \langle 0, 0, z^3/3 \rangle$

$x^2 + y^2 + z^2 = 1$

$\text{div}F = 0 + 0 + \frac{z^4}{12}$

$\iiint \frac{z^4}{12}\, dV$

$\int_0^{2\pi}\int_0^1 \frac{z^4}{12}\, dr\, d\theta$

$\frac{16\pi^4}{12}$

7. $F(x,y,z) = \langle xy^2, yz^2, zx^2 \rangle$

$x^2 + y^2 < 4 \Rightarrow r = 2$

$0 \leq z \leq 3$

$\text{div}F = y^2 + z^2 + x^2$

$\int_0^{2\pi}\int_0^2\int_0^3 (r^2\cos^2\theta + r^2\sin^2\theta + z^2)\, r\, dz\, dr\, d\theta$

inner int:

$r^3 z + \frac{rz^3}{3} \rightarrow \int_0^2 \left(r^3 z + \frac{rz^3}{3}\right)\Big|_0^3 dr$

$= \int_0^2 [r^3 + 3r]\, dr = \int_0^{2\pi} 30\, d\theta = 60\pi$

$\hookrightarrow \int_0^{2\pi}\int_0^2\int_0^2 (3x^2 + 3z^2)\, dz\, dr\, d\theta$

inner int:

$\int_0^2 3x^2 + 3z^2\, dz = [3x^2 z + z^3]\Big|_0^2$

$= 6x^2 + 8$

middle int:

$\int_0^2 (6r^2\cos^2\theta + 8)\, dr$

$= 2r^3\cos^2\theta + 8r\Big|_0^2 = 16\cos^2 + 16$

11. $F(x,y,z) =$

$\langle x^3, 0, z^3 \rangle$

$x^2 + y^2 + z^2 \leq 4$

$r = 2$

$0 \leq \theta \leq 2\pi$

$0 \leq z \leq 2$

$\text{div}F = \langle 3x^2 + 3z^2 \rangle$

outer int:

$\int_0^{2\pi} (16\cos^2\theta + 16)\, d\theta$

$= \left[8\left(\theta + \frac{1}{2}\sin(2\theta)\right) + 16\theta\right]\Big|_0^{2\pi}$

$= \frac{32\pi}{5}$

15. $F(x,y,z) = \langle x+y, z, z-x \rangle$

$z = 9 - x^2 - y^2$

$\operatorname{div} F = (1 + 0 + 1) = 2$

$\int_0^{2\pi} \int_0^3 \int_0^3 2 \, dz \, dr \, d\theta$

$= \int_0^3 6 \, dr = \int_0^{2\pi} 18 = 36\pi$?