17.3 Homework

1. $\iint_S F \cdot dS = \iiint_V div(F)\, dV$

$div(F) = 0$

$= \int_0^5 \int_0^2 \int_0^4 0 \, dV = \boxed{0}$

3. $\iint_S F \cdot dS = \iiint_V div(F)\, dV$

$div(F) = 2+0+0 = 2$ $\quad \iiint_V 2 \, dV$

$= (\text{volume})(\text{Integrand}) = (12r^2h)(2)$

$= 12(2)(2) = \boxed{48}$

5. $\iint_S F \cdot dS = \iiint_V div(F)\, dV$

$div(F) = z^2$ $\quad \iiint_V z^2 \, dV$

$0 \le \rho \le 1$
$0 \le \theta \le 2\pi$
$0 \le \varphi \le \pi$

$\int_0^{\pi} \int_0^{2\pi} \int_0^1 \rho^2 \cos^2\varphi \, \rho^2 \sin\varphi \, d\rho \, d\theta \, d\varphi$

$= \int_0^1 \rho^4 d\rho \int_0^{2\pi} d\theta \int_0^{\pi} \cos^2\varphi \sin\varphi \, d\varphi$

$u = \cos\varphi$
$du = -\sin\varphi \, d\varphi$

$= \left(\frac{1}{5}\right)(2\pi) \int_1^{-1} \frac{u^2 \sin\varphi}{-\sin\varphi} \, du = \left(\frac{2\pi}{5}\right) \int_{-1}^{1} u^2 \, du = \left(\frac{2\pi}{5}\right)\left(\frac{1}{3} + \frac{1}{3}\right)$

$= \boxed{\frac{4\pi}{15}}$

7. $\iint_S F \cdot dS = \iiint_V div(F)\, dV$

$div(F) = x^2 + z^2 + x^2$

$0 \le r \le 2$
$0 \le \theta \le 2\pi$
$0 \le z \le 3$

$\iiint (r^2 + z^2)\, r\, dz\, dr\, d\theta$

$= \int_0^{2\pi} \int_0^2 \int_0^3 r^3 + r z^2 \, dz\, dr\, d\theta$

$= \int_0^{2\pi} \int_0^2 zr^3 + \frac{z^3 r}{3} \Big|_0^3 \, dr\, d\theta$

$= \int_0^{2\pi} \int_0^2 3r^3 + 9r \, dr\, d\theta$

$= \int_0^{2\pi} \frac{3}{4} r^4 + \frac{9}{2} r^2 \Big|_0^2 \, d\theta$

$= \int_0^{2\pi} 12 + 18 \, d\theta$

$= (30)(2\pi) = \boxed{60\pi}$

11. $\iint_S F \cdot dS = \iiint_V div(F)\, dV$

$div(F) = 3x^2 + 3z^2$

$0 \le \rho \le 2$
$0 \le \theta \le \pi/2$
$0 \le \varphi \le \pi/2$

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{2} 3(\rho^2 - \rho^2 \sin^2\varphi \sin^2\theta)\, \rho^2 \sin\varphi \, d\rho\, d\theta\, d\varphi$$

$$= 3\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{2} \rho^4 (\sin\varphi - \sin^3\varphi \sin^2\theta)\, d\rho\, d\theta\, d\varphi$$

$$= \frac{48}{5}\int_0^{\pi/2}\int_0^{\pi/2} \sin\varphi - \sin^3\varphi \sin^2\theta \, d\theta\, d\varphi$$

$$= \boxed{\frac{32\pi}{5}}$$

15. $\iint_S F\, dS = \iiint_V div(F)\, dV$

$div(F) = 2$

$$\iiint_V 2\, dV = 2\iiint r\, dz\, dr\, d\theta$$

$0 \le z \le 9 - r^2$
$0 \le r \le 3$
$0 \le \theta \le 2\pi$

$$= 2\int_0^{2\pi}\int_0^{3}\int_0^{9-r^2} r\, dz\, dr\, d\theta$$

$$= 2\int_0^{2\pi}\int_0^{3} 9r - r^3\, dr\, d\theta$$

$$= 2\int_0^{2\pi} \frac{9}{2}r^2 - \frac{r^4}{4}\Big|_0^3 \, d\theta$$

$$= 2\int_0^{2\pi} \frac{81}{2} - \frac{81}{4} = 2\int_0^{2\pi} \frac{81}{4} = \left(\frac{81}{2}\right)(2\pi) = \boxed{81\pi}$$