Ex 17.3

~~1, 3, 5~~, 7, 11, 15

① $F(x,y,z) = \langle z, x, y \rangle$, the box $[0,4] \times [0,2] \times [0,3]$

$$\iint_S F \cdot ds = \iiint_R \operatorname{div} F \, dv = 0$$

③ $F(x,y,z) = \langle 2x, 3z, 3y \rangle$, the region $x^2+y^2 \le 1$, $0 \le z \le 2$

cylinderical co-ordinates

$$\boxed{\iint_S F \cdot ds = 2\int_0^{2\pi}\int_0^{1}\int_0^{2} dz\, dr\, d\theta}$$

$x^2+y^2 = r^2$

$r = 0$ to 1

$\theta = 0$ to 2π

$z = 0$ to 2

$$= \int_0^{2\pi}\int_0^{1} z\Big|_0^2 \, dr\, d\theta$$

$$= \int_0^{2\pi} 2r\Big|_0^1 \, d\theta$$

$$= \int_0^{2\pi} 2 \, d\theta$$

$$= 2\theta\Big|_0^{2\pi} \qquad = 4\pi$$

⑤ $F(x,y,z) = \left\langle 0, 0, \frac{z^3}{3} \right\rangle$, S is the sphere $\boxed{x^2+y^2+z^2=1}$

spherical coordinates

$$\int_0^{\pi}\int_0^{2\pi}\int_0^{1} z^2 (\rho^2 \sin\phi)\, d\rho\, d\theta\, d\phi$$

$$\int_0^{\pi}\int_0^{2\pi}\int_0^{1} \rho^4 \cos^2\phi \sin\phi \quad d\rho\, d\theta\, d\phi$$

$$= \frac{2\pi}{15} - \frac{2}{15}(-1)^3\pi$$

$$= \frac{2\pi}{15} + \frac{2\pi}{15}$$

$$\boxed{= \frac{4\pi}{15}}$$

(7) $F(x,y,z) = \langle xy^2, yz^2, zx^2 \rangle$, where $S =$
$x^2+y^2 \le 4$, $0 \le z \le 3$
Cylinderical

$$\iiint (y^2 + z^2 + x^2)\, dV$$

$$= \int_0^{2\pi}\int_0^{2}\int_0^{3} (r^2+z^2) r \, dr\, dz\, d\theta$$

$$\boxed{= 60\pi}$$

$x^2+y^2 = r^2$

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

$$x^2+y^2 = r^2\cos^2\theta + r^2\sin^2\theta$$
$$= r^2(1)$$
$$= r^2$$

(11) $\iiint 3x^2 + 3z^2 \, dV$

$x^2+y^2+z^2 \le 4$
$x \ge 0$
$y \ge 0$
$z \ge 0$

$$3\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{2} \rho^2\cos^2\phi + \rho^2\sin^2\phi\cos^2\theta$$

$$= \frac{32\pi}{5}$$

(15) $F(x,y,z) = \langle x+y, z, z-x \rangle$

$$\boxed{z = 9-x^2-y^2}$$

$$\iiint (1+1)\, dV$$

$$2\int_0^{9-r^2}\int_0^{2\pi}\int_0^{3} r \, dr\, d\theta\, dz$$

$$\boxed{= 81\pi}$$

$z = 9-x^2-y^2$
$z = 9-r^2$
$x = r\cos\theta$
$y = r\sin\theta$

$0 = 9 - r^2$