Calc III Exam 1

1) $x + y^9 + z^3 + xyz^2 = 4$, Find $\frac{\partial z}{\partial y}$ at $(1,1,1)$

$$\frac{\partial z}{\partial y} = \frac{-F_y}{F_z}$$

$F_y = 9y^8 + xz^2$

$F_z = 3z^2 + 2xyz$

$$\frac{\partial z}{\partial y} = -\frac{(9y^8 + xz^2)}{3z^2 + 2xyz}$$

at $(1,1,1)$, plug in

$$\frac{\partial z}{\partial y} \text{ at } (1,1,1) = -\frac{(9(1)^8 + (1)(1)^2)}{3(1)^2 + 2(1)(1)(1)} = -\frac{(10)}{5} = \boxed{-2}$$

2) Suppose that $\vec{\nabla} f(P) = \langle 1,-1,4\rangle$ Is f increasing or decreasing at the Direction $\langle 1,3,-1\rangle$?

$\vec{v} = \langle 1,3,-1\rangle$

$\|\vec{v}\| = \sqrt{1^2+3^2+1^2} = \sqrt{11}$

$\vec{u}_v = \langle \frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}}, \frac{-1}{\sqrt{11}} \rangle$

$D_{\vec{u}} f(P) = \vec{\nabla} f_P \cdot \vec{u}$

$= \langle 1,-1,4\rangle \cdot \langle \frac{1}{\sqrt{11}}, \frac{3}{\sqrt{11}}, \frac{-1}{\sqrt{11}} \rangle$

$= \frac{1}{\sqrt{11}} - \frac{3}{\sqrt{11}} - \frac{4}{\sqrt{11}} = -\frac{6}{\sqrt{11}} \rightarrow$ decreasing

3) $3x^3 + 3y^3 + 7z^3$, $P(1,-1,1)$, $Q = \langle 1,-1,3\rangle$

$\vec{v} = PQ = \langle 0,-2,2\rangle$

$f_x = 9x^2$

$f_y = 9y^2$

$f_z = 21z^2$

$\vec{\nabla} f = \langle 9x^2, 9y^2, 21z^2\rangle$

$\vec{\nabla} f(1,-1,1) = \langle 9,-9,21\rangle$

$\|\vec{v}\| = \sqrt{0^2+2^2+2^2} = \sqrt{8}$

$\vec{u}_v = \langle 0, -\frac{2}{\sqrt{8}}, \frac{2}{\sqrt{8}} \rangle$

$D_{\vec{u}} f(1,-1,1) = \langle 9,-9,21\rangle \cdot \langle 0, -\frac{2}{\sqrt{8}}, \frac{2}{\sqrt{8}} \rangle$

$= 0 + \frac{18}{\sqrt{8}} + \frac{42}{\sqrt{8}}$

$= \frac{60}{\sqrt{8}}$

6) $\lim_{(x,y)\to(1,1)} \frac{x^2-y^2}{x-y}$: $\frac{0}{0}$ when plugged in

$\lim_{(x,y)\to(1,1)} \frac{\cancel{(x-y)}(x+y)}{\cancel{x-y}} = \lim_{(x,y)\to(1,1)} x+y = 2$

7) $r(t) = \langle 1, 9t, 3t^2 \rangle$ at $(1,0,0)$

$r'(t) = \langle 0, 9, 6t \rangle$

$r''(t) = \langle 0, 0, 6 \rangle$

$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 0 & 9 & \cancel{6t} \\ 0 & 0 & 6 \end{vmatrix} = (54-0)i - (0-0)j + (0-0)k$

$= 54i$

$\|r'(t)\| = \sqrt{0^2+9^2+0^2} = 9$ at $(1,0,0)$

$k(t) = \frac{\|r'(t) \times r''(t)\|}{\|r'(t)\|^3} = \frac{54}{9^3} = \frac{54}{729}$

8) $r''(t) = \langle -\sin(t), -9\cos t \rangle$, $t=0$, position is $(0,9)$, vel is $(1,0)$.

$\int r''(t) = \langle \cos t, -9\sin t \rangle = r'(t)$

$\int r'(t) = \langle \sin t, 9\cos t \rangle = r(t)$

$r(\pi) = \langle \sin(\pi), 9\cos(\pi) \rangle$

$r(\pi) = \langle 0, -9 \rangle$, $\boxed{(0,-9)}$

9) rate of change wrt $x = 1$ $\frac{df}{dx}$

wrt $y = 2$ $\frac{df}{dy}$

$\frac{dx}{dt} = 1$

$\frac{dy}{dt} = 8$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$= 1(1) + 2(8) = 1+16 = \boxed{17}$$

10) $x=1$, $y=2$, $z=3$ find point of intersection.

(1,2,3).

4) $e^{x-1} - (x-1)e^{y-3}$

$f_x = e^{x-1} - e^{y-3} = 0$ $x=1$, $y=3$

$f_y = -e^{y-3}(x-1) = 0$

CP: (1,3)

$f_{xx} = e^{x-1}$

$f_{xy} = -e^{y-3}$

$f_{yy} = -e^{y-3}(x-1)$

$D = (e^{x-1})(-e^{y-3}(x-1)) - (-e^{y-3})^2$

at (1,3) $= e^0(-e^0(0) - (-e^0)^2$

$= 1(0-1) = 1(-1) = \boxed{-1}$ $D=-1$

$f_{xx} = 1$

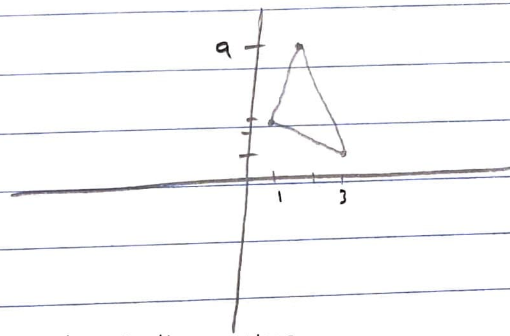
sign of D	sign of f_{xx}	
−	m	saddle point

5) $f(x,y) = x + 2y + 9$

$A(1,9)$

$B(3,1)$

$C(1,3)$



Since it is linear finalists are automatically vertices.

$f(1,9) = 1 + 2(9) + 9 = 28$

$f(3,1) = 3 + 2(1) + 9 = 14$

$f(1,3) = 1 + 2(3) + 9 = 16$

$\boxed{\text{global min} = 14}$

$f_x = 1$

$f_y = 2$