MIDTERM 1 FOR Multivariable Calculus, Math 251(22-24), FALL 2020, Dr. Z.

 NAME:

Orion Kress Sanfilippo

 RUID:

204006944

 EMAIL:

omk23@scarletmail.rutgers.edu

 BELOW WRITE THE LIST OF THE ANSWERS

 Answer[ 1 ]= $\frac{-1}{3}$

 Answer[ 2 ]= f is decreasing

 Answer[ 3 ]= 12

 Answer[ 4 ]= P = (1,6)

 Answer[ 5 ]= 12

 Answer[ 6 ]= 1

 Answer[ 7 ]= 8

 Answer[ 8 ]= P = (0,1)

 Answer[ 9 ]= 38

 Answer[ 10 ]= (1,9,4)

 -----------------------------------------------------------------

 Instructions: Download this file with its original name, mt1.txt, then rename it, in your computer

 mt1FirstLast.txt

 Edit it with your answers and solutions

USING COMPUTEREZE: e.g.: x times y IS x\*y, x to the power y is x^y

 and Email DrZcalc3@gmail.com, 80 minutes (or sooner) after starting (for most people 10:00am, Oct. 15)

 Subject: mt1

 with an attachment. YOU MUST NAME IT EXACTLY

 mt1FirstLast.txt

 ---------------------------------------------------------------------------

 For each of the questions you MUST first figure, YOUR version, with the following convention

 For i=1,2,3,4,5,6,7,8,9 , a[i]:= The i-th digit of your RUID, BUT of it is zero make it 1

 Example: RUID=413200125;

 a[1] = 4, a[2] = 1, a[3] = 3, a[4] = 2, a[5] = 1, a[6] = 1, a[7] = 1, a[8] = 2, a[9] = 5

-----------------------------------------------------------------------------------------------------------------------------------------------

 HERE WRITE THE ACTUAL a[i]

 a[1]= 2, a[2]= 1, a[3]= 4, a[4]= 1 , a[5]= 1, a[6]= 6, a[7]= 9, a[8]= 4, a[9]= 4

 --------------------------------------------

 ---------------------------------------------

 Problem 1:

 Find dz/dy at the point (1,1,1) if z(x,y) is given implicitly by the equation

x^a[1]+y^a[2]+z^a[3]+a[5]\*x\*y\*z^2 = 3+a[5]

 With my RUID data the question is

 Find dz/dy at the point (1,1,1) if z(x,y) is given implicitly by the equation

x^2+y^1+z^4+1\*x\*y\*z^2 = 3+1

 Here is how I do it (Explain everything)

Have to find derivative of z with respect to y, start by using the prime

(x^2+y^1+z^4+1\*x\*y\*z^2 = 3+1)’ (w.r.t. y) =

0 + 1 + 4\*z^3\*z’ + x(z^2 + 2\*y\*z\*z’) = 0

Isolate terms with z’ on one side:

z’(4\*z^3 + 2\*x\*y\*z) = -x\*z^2 - 1

Then divide:

dz/dy = $\frac{(-x \*z^{2} - 1)}{(4\*z^{3}+ 2\*x\*y\*z)}$

To find derivative at a point, plug in values:

@ P = (1,1,1), dz/dy = $\frac{(-1 \*1^{2} - 1)}{(4\*1^{3}+ 2\*1\*1\*1)}= \frac{-2}{6}=\frac{-1}{3}$

 Ans.:

dz/dy = $\frac{-1}{3}$

 ---------------------------------------------

 Problem 2:

 Suppose that grad(f)(P)=<a[1],-a[4],a[7]+2>. Is f increasing or decreasing at the direction <a[1],a[3],-a[5]>?

 With my RUID data the question is

Suppose that grad(f)(P)=<2,-1,11>. Is f increasing or decreasing at the direction <2,4,-1>?

 Here is how I do it (Explain everything)

In order to determine whether f(x,y,z) is increasing or decreasing *in a certain direction*, we have to use dot product with the direction we are given:

(grad(f)(P)) ૦ <2,4,-1> =

(2\*2 + -1\*4 + -1\*11) = 4 - 4 - 11 = **-11**

**Since the dot product is negative, f is not increasing in this direction.**

 Ans.:

f is decreasing in the direction (2,4,-1)

 ---------------------------------------------

 Problem 3:

 Find the directional derivative of the function f(x,y,z)

x^3\*a[6]+y^3\*a[3]+z^3\*a[8]

 At the point P=(1,-1,1) in the direction pointing to Q=(1,-1,3)

 With my RUID data the question is

 Find the directional derivative of the function f(x,y,z)

x^3\*6+y^3\*4+z^3\*4

 At the point P=(1,-1,1) in the direction pointing to Q=(1,-1,3)

 Here is how I do it (Explain everything)

First, find gradient of f:

grad(f(x,y,z)) = < 18x^2, 12y^2, 12z^2 >

Plug in x\_0, y\_0 and z\_0

grad(f(1,-1,1)) = < 18, 12, 12 >

To find directional derivative, take dot product of grad(f(P)) and provided direction, BUT we need to find and normalize direction vector **first**:

PQ = (1 - 1, (-1) - (-1), 3 - 1)/|PQ| =

PQ = ( 0 , 0 , 1)

Now take dot prod:

<18,12,12> ૦ <0,0,1> =

12

 Ans.:

The directional derivative of f at point P in the direction towards Q is 12.

 ---------------------------------------------

 Problem 4:

 Find a saddle point of the function f(x,y)=

exp(x-a[4])-(x-a[4])\*exp(y-a[6])

 If there is no saddle point, write in the Answers: "Does Not exist". Explain what you are doing

 With my RUID data the question is

 Find a saddle point of the function f(x,y)=

exp(x-1)-(x-1)\*exp(y-6)

 If there is no saddle point, write in the Answers: "Does Not exist". Explain what you are doing.

 Here is how I do it (Explain everything)

First find the partial derivatives of f:

df/dx = exp(x-1) - exp(y-6)

df/dy = -(x-1)\*exp(y-6)

Then create a system of equations, set partial derivatives equal to 0 and solve:

df/dx = exp(x-1) - exp(y-6) = 0

df/dy = -(x-1)\*exp(y-6) = 0

In df/dy, (exp(y-6)) can never equal 0, therefore

1-x = 0, **x = 1**

Plugging into df/dx,

exp(1-1) = exp(y-6)

1 = exp(y-6)

ln(1) = y - 6,

**y = 6**

To test point categorization, find second derivatives:

d^2f/dx^2 = exp(x-1)

d^2f/dy^2 = -(x-1)exp(y-6)

d^2f/dx\*dy = -exp(y-6)

Next, plug in P = (1, 6) found in previous part and find the discriminant:

D = (d^2f/dx^2)\*(d^2f/dy^2) - (d^2f/dx\*dy)^2

= (exp(1-1)\*-(1-1)\*exp(6-6)) -

= 0 - 1

= **-1**

Since D< 0, P = (1,6) is this function’s **only** saddle point

 Ans.:

P = (1,6) is the only saddle point.

 ---------------------------------------------

 Problem 5:

 Let f(x,y) be the function

a[4]\*x + a[7]\*y + a[2]

 Find the ABSOLUTE MINIMUM VALUE of f(x,y) INSIDE the TRIANGLE whose VERTICES ARE

A = [a[1], a[2]], B = [a[3], a[4]], C = [a[5], a[6]]

 With my RUID data the question is

 Let f(x,y) be the function

1\*x + 9\*y + 1

 Find the ABSOLUTE MINIMUM VALUE of f(x,y) INSIDE the TRIANGLE whose VERTICES ARE

A = [2, 1], B = [4, 1], C = [1, 6]

 Here is how I do it (Explain everything)

Must find extreme values and all critical points of f **within the triangular interval:**

df/dx = 1

df/dx = 9

Since the partial derivatives are constants, all extreme values can be found by plugging in the bounds of the area, then checking between these values

f(A) = 1\*2 + 9\*1 + 1 = 12

f(B) = 1\*4 + 9\*1 + 1 = 14

f(C) = 1\*1 + 9\*6 + 1 = 56

Since f is linear along each side of the triangle, no need to check intermediate values, the absolute min of f on this interval exists at f(A), and is equal to 12

 Ans.:

The absolute minimum value of f on this interval is 12, and occurs at point A.

 ---------------------------------------------

 Problem 6:

 Let f(x,y) be the function

(x^2\*a[4]^2-y^2\*a[5]^2)/(x\*a[4]-y\*a[5])

 Find the LIMIT of f(x,y) as (x,y) goes to the point [a[5],a[4]], or show that it does not exist

 With my RUID data the question is

 Let f(x,y) be the function

(x^2\*1^2-y^2\*1^2)/(x\*1-y\*1)

 Find the LIMIT of f(x,y) as (x,y) goes to the point [1,1], or show that it does not exist

 Here is how I do it (Explain everything)

$lim f(x,y)\_{x->1,y->1}(\frac{x^{2}-y^{2}}{x - y}) = L$

Plugging in values here is trivial and unnecessary, clearly yields 0/0.

Must test what happens when we approach 1,1 along the line y = cx:

$lim f(x,y)\_{x->1,cx->1}(\frac{x^{2}-cx^{2}}{x - cx}) =lim f(x,y)\_{x->1,cx->1}\frac{x^{2}(1-c)}{x(1-c)} = $

$=lim f(x,y)\_{x->1,cx->1} x=1 $

Since this limit is not dependent on the slope of the line on which we approach, the limit of f as x and y both approach 1 is **1**

Ans:

 The limit of f as x and y both approach 1 is **1.**

 ---------------------------------------------

 Problem 7:

 Find the curvature of the curve

r(t) = [a[1], a[2]\*t, a[3]\*t^2]

 At the point (a[1],0,0)

 With my RUID data the question is

 Find the curvature of the curve

r(t) = [2, 1\*t, 4\*t^2]

 At the point (2,0,0)

 Here is how I do it (Explain everything)

First find the prime and double prime of r:

r’ = [0, 1, 8t]

r’’ = [0,0,8]

The formula for curvature is

k = $\frac{|r'(t) x r"(t)|}{|r'(t)|^{3}}$

|r’ x r’’| = |i(8 - 0) - j(0) - k(0) | = 8

|r’(t)|^3 @ P = (2,0,0) = $(\sqrt{0^{2}+1^{2}+8(0}))^{3}=1$

Ans:

The curvature at this point is k = 8

 ---------------------------------------------

 Problem 8:

 A particle is moving in the plane with ACCELERATION given by

[-a[1]\*sin(t), -a[2]\*cos(t)]

 At time t=0 its position is , [0, a[2]]

 and its velocity is , [a[1], 0]

 Where is it located at time , t = Pi

 With my RUID data the question is

 Problem 8:

 A particle is moving in the plane with ACCELERATION given by

[-2\*sin(t), -1\*cos(t)]

 At time t=0 its position is , [0, 1]

 and its velocity is , [2, 0]

 Where is it located at time , t = Pi

 Here is how I do it (Explain everything)

In order to go from the acceleration to the velocity and then position, take 2 integrals:

$v(t) = \int\_{}^{}a(t)dt =\int\_{}^{}<-2\*sin(t), -1\*cos(t)>dt$

$v(t) = <2cos(t), -1sin(t)> + C$

$v(0) = <2,0>, C=0$

$r(t) =$$\int\_{}^{}v(t)dt = <2sin(t), cos(t)>$

$r(0) = <0,1>, C=0$

$r(t) = $$ <2sin(t), cos(t)>$

$ r(pi) = <2sin(pi), cos(pi)> = (0, -1)$

Ans:

The position at this time is r(pi) = **(0,1)**

 ---------------------------------------------

 Problem 9:

 A certain function depends on variables x and y

 Right now the rate of change of the function with respect to x is, a[5]

 and the rate of change of the function with respect to y is, a[7]

 Both x and y depend on time

 Right now the rate of change of x with respect to time is, a[1]

 and the rate of change of y with respect to time is, a[9]

 How fast is the function changing right now?

  **With my RUID data the question is**

 A certain function depends on variables x and y

 Right now the rate of change of the function with respect to x is, 1

 and the rate of change of the function with respect to y is, 9

 Both x and y depend on time

 Right now the rate of change of x with respect to time is, 2

 and the rate of change of y with respect to time is, 4

 How fast is the function changing right now?

 Here is how I do it (Explain everything)

Since f is dependent on x and y, AND s and y are both dependent on t, we must use the chain rule to determine the derivative of f with respect to t:

df/dt = df/dx\*dx/dt + df/dy\*dy/dt

df/dx = 1, dx/dt = 2, df/dy = 9, dy/dt = 4

df/dt = 1\*2 + 9\*4 = 38

Ans:

The speed of the function at the given point **with respect to t** is 38.

 ---------------------------------------------

 Problem 10:

 Find the point of intersection of the three planes

 x = a[5], y = a[7], z = a[3]

 With my RUID data the question is

 Find the point of intersection of the three planes

 x = 1, y = 9, z = 4

 Here is how I do it (Explain everything)

Since each of these shapes are flat and must intersect at at least one point, the point corresponds to the value of each plane:

P = (1,9,4)

Ans:

(1,9,4)