

MT1 Work

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8. $A = [-\sin(t), -9\cos(t)]$

$t=0 \Rightarrow$ position is $(0, 9)$

\Rightarrow velocity is $(1, 0)$

$$V = [\cos(t), -9\sin(t)]$$

$$P = [\sin(t), 9\cos(t)]$$

$t = \pi$

$$P(\pi) = [\sin(\pi), 9\cos(\pi)] = (0, -9)$$

9. $dz/dx = 0$

$$dx/dt = 1$$

$$dz/dy = 6$$

$$dy/dt = 1$$

$$\frac{dz}{dt} = \left(\frac{dz}{dx}\right)\left(\frac{dx}{dt}\right) + \left(\frac{dz}{dy}\right)\left(\frac{dy}{dt}\right) = 0(1) + 6(1) = 6$$

7. $r(t) = [1, 9t, 3t^2]$ at pt. $(1, 0, 0)$.

$$r'(t) = (0, 9, 6t)$$

$$r''(t) = (0, 0, 6)$$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 0 & 9 & 6t \\ 0 & 0 & 6 \end{vmatrix} = i \begin{vmatrix} 9 & 6t \\ 0 & 6 \end{vmatrix} - j \begin{vmatrix} 0 & 6t \\ 0 & 6 \end{vmatrix} + k \begin{vmatrix} 0 & 9 \\ 0 & 0 \end{vmatrix}$$

$$= 54i$$

$$|r'(t) \times r''(t)| = \sqrt{54^2} = 54$$

$$|r'(t)| = \sqrt{9^2 + (6t)^2} = \sqrt{81 + 36t^2}$$

$$K(t) = \frac{54}{(\sqrt{81 + 36t^2})^3}$$

6. $f(x, y) = \frac{0}{0-0} = 0$

2. $\nabla f_p = \langle 1, 0, 8 \rangle$ If f incr. or decr. at direction $v = \langle 1, 3, 0 \rangle$

$$D_v f(p) = \nabla f_p \cdot v = 1 + 0 + 0 = 1 > 0$$

since deriv. is (+), f is incr. at P in direction v .

5. $f(x, y) = 6y + 9$
 $A = [(1, 9), (3, 0), (0, 0)]$

$$f_x = 0$$

$$f_y = 6$$

4. $f(x, y) = e^x - xe^y$
 $f_x = e^x - e^y = 0$ $(0, 0)$ crit. pt.
 $f_y = -xe^y = 0$

$$f_{xx} = e^x$$

$$f_{xy} = -e^y$$

$$f_{yy} = -xe^y$$

$$D = e^x(-xe^y) - (-e^y)(-e^y)$$

$$= -xe^x e^y - e^y e^y$$

$$= -xe^{x+y} - e^{2y}$$

$$D_{(0,0)} = -1 < 0$$

3. $f(x, y, z) = 3y^3 + 2z^3$ at pt. $(1, -1, 1)$ in direction $Q = (1, -1, 3)$

$$\nabla f = \langle 0, 9y^2, 6z^2 \rangle$$

$$\nabla f(1, -1, 1) = \langle 0, 9, 6 \rangle$$

$$| \langle 1, -1, 3 \rangle | = \sqrt{1^2 + (-1)^2 + 3^2} = \sqrt{11}$$

$$u = \left\langle \frac{1}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{3}{\sqrt{11}} \right\rangle$$

$$\nabla f \cdot u = \langle 0, 9, 6 \rangle \cdot \left\langle \frac{1}{\sqrt{11}}, \frac{-1}{\sqrt{11}}, \frac{3}{\sqrt{11}} \right\rangle$$

$$= 0 + \left(\frac{-9}{\sqrt{11}} \right) + \left(\frac{18}{\sqrt{11}} \right) = \frac{9}{\sqrt{11}} = \frac{9\sqrt{11}}{11}$$

1. dz/dy at pt. $(1, 1, 1)$ of $z(x, y) \Rightarrow x + y^9 + z^3 = 3$
 $F(x, y, z) = x + y^9 + z^3 - 3$

$$\frac{dz}{dy} = -\frac{F_y}{F_z}$$

$$F_y = 9y^8$$

$$F_z = 3z^2$$

$$\Rightarrow -\frac{9y^8}{3z^2} = -\frac{3y^8}{z^2}$$