No.

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SCC: I and II

Final answer:

Q1: -18

Q2: $\int_0^1 \int_{y^2}^1 f(x,y)\, dx\, dy$

Q3: $z = -3\sqrt{3}\,x - 5\sqrt{3}\,y + \frac{8\sqrt{3}\pi + \pi}{6}$

Q4: $-\mathbf{i} - 8\mathbf{j} + 5\mathbf{k}$

Q5: A: $60°$ $\frac{\pi}{3}$ B: $60°$ $\frac{\pi}{3}$ C: $60°$ $\frac{\pi}{3}$

Q6: $-4\sqrt{3}$

Q7: -1.807

Q8: 8π

Q9. -15

Q10: $f(\frac{3}{4}, -1)$ local minimum

Q11: ~~3.0000667~~ 3.000333

Q12: $\frac{1}{3}$

Q13: $\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{\pi} \int_0^2 (\rho \sin\phi \cos\theta)^2 (\rho \sin\phi \sin\theta)(\rho\cos\phi)\rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$

Q14: $\frac{1}{3}$

Q15: $\int_0^1 \int_0^v (u^2 + uv + v^2)\, du\, dv$

Q16: 14

Q17: 0 (exist).

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I hereby declare that all works were done by myself. I was allowed to use Maple, calculators, the book and all the material in the web-page of this class but not other resources on the internet.

I only spent 3 hours on doing the exam. The last 30 minutes were spent in checking & double checking the answer.

I also understand that I may be subject to a random short chat to verify that I actually did it all by myself.

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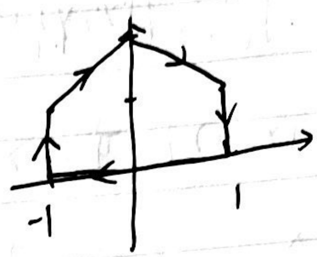
Q1. $\frac{dQ}{dx} = 11 \quad \frac{dP}{dy} = 5$



$\int_C P\,dx + Q\,dy$

$= \iint_D (11-5)\,dA$

$= 6 \times (3)$

$= 18$

because it is clockwise, it is negative direction.

$\therefore$ it is equal to -18.

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Q7. $\int_{\frac{1}{4}}^{1} \int_{0}^{\sqrt{x}} f(x,y)\,dy\,dx$

Ans: $\{(x,y) \mid \frac{1}{4} \leq x \leq 1 \quad 0 \leq y \leq \sqrt{x}\}$.

$y = \sqrt{x}$ $\quad \therefore$ type 2:

$x = y^2$ $\quad \{(x,y) \mid y^2 \leq x \leq 1 \quad 0 \leq y \leq 1\}$.

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Q3.

Ans: $f_x = -2\sin(x+y) - 4\sin(x+z) = 0$

$f_y = -2\sin(x+y) - 8\sin(y+z) = 0.$

$f_x(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}) = -\sqrt{3} - 2\sqrt{3} = -3\sqrt{3}$

$f_y(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}) = -\sqrt{3} - 4\sqrt{3} = -5\sqrt{3}$

$$z - \frac{\pi}{6} = -3\sqrt{3}(x - \frac{\pi}{6}) + (-5\sqrt{3})(y - \frac{\pi}{6})$$

$$z = -3\sqrt{3}x - 5\sqrt{3}y + \frac{\sqrt{3}\pi}{2} + \frac{5\sqrt{3}\pi}{6} + \frac{\pi}{6}$$

$$z = -3\sqrt{3}x - 5\sqrt{3}y + \frac{8\sqrt{3}\pi + \pi}{6}.$$

Q4. $(a+b+c) \times (2a-b+3c)$

$$= \begin{vmatrix} a, & b, & c \\ 2a, & -b, & 3c \end{vmatrix} = i(3bc+bc) - j(3ac-2ac) + k(-ab-2ab)$$

$$= 4bc\,i - ac\,j - 3ab\cdot k$$

$$= 4(i-j+k) - (2i+j+2k) - 3(i+j-k)$$

$$= -i - 8j + 5k$$

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Q5. I draw the graph on Maple, its a equilateral triangle

the angle at A is 60°, radians: $\frac{\pi}{3}$

the angle at B is 60°, radians: $\frac{\pi}{3}$

the angle at C is 60°, radians: $\frac{\pi}{3}$

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Q6. $P=(1,1,1)$ $Q=(-1,-1,-1)$

$PQ=(-2,-2,-2)$

$f_x = 3x^2+yz$ $f_y = 3y^2+xz$ $f_z = 3z^2+xy$.

$\|(-2,-2,-2)\| = \sqrt{12}$

$u = \left(\frac{-2}{\sqrt{12}}, \frac{-2}{\sqrt{12}}, \frac{-2}{\sqrt{12}}\right)$

$\nabla f = (4,4,4)$

$\nabla f \cdot u = \left(\frac{-8}{\sqrt{12}}, \frac{-8}{\sqrt{12}}, \frac{-8}{\sqrt{12}}\right) = \frac{-24}{\sqrt{12}} = -4\sqrt{3}$.

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Q7. $\frac{dg}{du} = ?$

Ans: $\frac{dg}{du} = \frac{dg}{dx} \cdot \frac{dx}{du} + \frac{dg}{dy} \cdot \frac{dy}{du}$

$= 6x \cdot e^{u} \cos v + (-6y) \cdot e^{u} \sin v$

$= 6(e^{u} \cos v) \cdot (e^{u} \cos v) - 6 (e^{u} \sin v) \cdot (e^{u} \cdot \sin v)$

$= 6 \cdot (e^{0} \cos 1)^2 - 6(e^{0} \sin 1)^2$

$= 6 (\cos 1^2 - \sin 1^2)$

$= -1.807$

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Q8.

$\operatorname{div} F = (3, -2, 5) = 6$

$\{(x,y,z) \mid 0 \leq x \leq 2,\ -\sqrt{4-x^2} \leq y \leq 0,\ 0 \leq z \leq \sqrt{4-x^2-y^2}$

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{0} \int_0^{\sqrt{4-x^2-y^2}} 6 \; dz\, dy\, dx$$

$$= \int_0^2 \int_{-\sqrt{4-x^2}}^{0} 6(\sqrt{4-x^2-y^2}) \; dy\, dx$$

$$= \int_0^2 4\pi \; dx$$

$$= 8\pi.$$

the answer is 8π.

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Q9. $\frac{dg}{dx} = 2 \quad \frac{dg}{dy} = 3$

$$\int_0^1 \int_0^1 -3z \cdot 2 - 2x(3) + y + z \; dy\, dx$$

$$= \int_0^1 \int_0^1 -6(2x+3y) - 6x + y + (2x+3y) \; dy\, dx$$

$$= \int_0^1 \int_0^1 -12x - 18y - 6x + y + 2x + 3y \; dy\, dx$$

$$= \int_0^1 \int_0^1 \cancel{-18}\; -16x - 14y \; dy\, dx.$$

$$= \int_0^1 -16xy - 7y^2 \Big|_0^1 dx$$

$$= \int_0^1 -16x - 7 \; dx$$

$$= -8x^2 - 7x \Big|_0^1 = -8 - 7 = -15.$$

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Q10. $f_x = 4 - \frac{2}{2x+y}$ $\qquad$ $f_y = -2y - \frac{1}{2x+y}$

$f_{xx} = \frac{4}{(2x+y)^2}$ $\qquad$ $f_{yy} = -2 + \frac{1}{(2x+y)^2}$

$f_{xy} = \frac{2}{(2x+y)^2}$

$4 - \frac{2}{2x+y} = 0$ $\qquad$ $-2y - \frac{1}{2x+y} = 0$

$4 = \frac{2}{2x+y}$ $\qquad$ $-2y = \frac{1}{2x+y}$

$x = \frac{3}{4}$ $\qquad$ $y = -1$

$f_{xx} = 16$ $\qquad$ $f_{xy} = 8$ both > 0

$f_{yy} = 6$

$D = f_{xx} f_{yy} - f_{xy}^2 = 32 > 0$

$\therefore$ local minimum at $f(\frac{3}{4}, -1)$

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Q11. $f_x = \dfrac{2x}{\sqrt{2x^2+3y^2+z^2}} = \dfrac{2}{3}$ $\qquad f_z = \dfrac{z}{\sqrt{2x^2+3y^2+z^2}} = \dfrac{2}{3}$

$f_y = \dfrac{3y}{\sqrt{2x^2+3y^2+z^2}} = 1$ $\qquad \sqrt{2+3+4} = 3$

$L(x,y,z) = \frac{2}{3}(x-1) + (y-1) + \frac{2}{3}(z-2) + 3$

$= 3.0003333$

Q12: $\displaystyle\int_0^{\frac{\sqrt{2}}{2}} \int_0^{x} x\,dy\,dx + \int_{\frac{\sqrt{2}}{2}}^{1} \int_0^{\sqrt{1-x^2}} x\,dy\,dx$

$\Rightarrow$ θ is from 0 to $\frac{\pi}{2}$

r is from $\frac{\sqrt{2}}{2}$ to 1

$\displaystyle = \int_0^{\frac{\sqrt{2}}{2}} xy\Big|_0^x\,dx + \int_0^{\frac{\pi}{2}} \int_{\frac{\sqrt{2}}{2}}^{1} r\cos\theta \cdot r\,dr\,d\theta$

$\displaystyle = \frac{\sqrt{2}}{12} + \int_0^{\frac{\pi}{2}} \frac{r^3}{3}\cos\theta\Big|_{\frac{\sqrt{2}}{2}}^{1}\,d\theta$

$\displaystyle = \frac{\sqrt{2}}{12} + \int_0^{\frac{\pi}{2}} \left(\frac{1}{3} - \frac{\sqrt{2}}{12}\right)\cos\theta\,d\theta$

$= \frac{\sqrt{2}}{12} + \left(\frac{1}{3} - \frac{\sqrt{2}}{12}\right)\left(\sin\frac{\pi}{2} - \sin 0\right)$

$= \frac{\sqrt{2}}{12} + \left(\frac{1}{3} - \frac{\sqrt{2}}{12}\right) \cdot 1$

$= \frac{1}{3}$

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Q13. ρ is from 0 to 2.

$\sqrt{4-z^2} = y$ it is from $\frac{3}{2}\pi$ to $\frac{\pi}{2}$ $y>0$ $\therefore 0$ to $\frac{\pi}{2}$

$x = \sqrt{-4-z^2-y^2}$ also $x<0$, $\therefore \theta$ is from $\frac{\pi}{2}$ to π.

$\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{\pi} \int_0^2 (\rho \sin\phi \cos\theta)^2 \cdot (\rho \sin\phi \sin\theta) \cdot \rho \cos\phi \cdot \rho^2$

$\sin\phi) \cdot d\rho \, d\theta \, d\phi$

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Q14: $r'(t) = (0, 3\cos t, -3\sin t) = (0, \frac{3}{2}, \frac{-3\sqrt{3}}{2})$

$r''(t) = (0, -3\sin t, -3\cos t)$

$$\begin{vmatrix} 0 & 3\cos t & -3\sin t \\ 0 & -3\sin t & -3\cos t \end{vmatrix}$$

$= i(-9\cos t^2 - 9\sin t^2) - 0j - 0k$ $\quad |r'(t)|$

$= (-9\cos t^2 - 9\sin t^2, 0, 0)$ $\quad = 3$

$= (-\frac{9}{4} - \frac{27}{4}, 0, 0) = (-9, 0, 0)$

$|r'(t) \times r''(t)| = 9$

$k(t) = \frac{9}{3^3} = \frac{1}{3}$

the curvature is $\frac{1}{3}$.

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Q15. $\cancel{\int_0^1 \int_0^u (u^2+uv+v^2)\,dv\,du}$

$$\int_0^1 \int_0^v (u^2+uv+v^2)\,du\,dv$$

Q16. $\text{grad}(f) = (y^2z^3,\ 2xyz^3,\ 3xy^2z^2) = (1,2,3)$

$\text{grad}(g) = (1,\ 2y,\ 3z^2) = (1,2,3)$

$$\text{grad}(f)\cdot\text{grad}(g) = (1,2,3)\cdot(1,2,3)$$
$$= (1+4+9)$$
$$= 14$$

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Q17. $y=mx \quad w=kz$

$$\frac{(x+mx)^2-(z+kz)^2}{(x+mx)-(z+kz)} =$$ set $x+mx=a$

$z+kz=b$

$$\frac{a^2-b^2}{a-b}=\frac{(a+b)(a-b)}{a-b}=a+b$$

$$= x+mx+z+kz$$

$$= x(1+m)+z(1+k)$$

$$= (x+y)+(z+w)$$

$$=0+0+0+0$$

$$=0$$

the limit is 0.

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