

5. $\vec{AB} = \langle 1, 0, 1 \rangle$

$\vec{AC} = \langle 1, 1, 0 \rangle$

$\vec{BC} = \langle 0, 1, -1 \rangle$

$A = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$B = \cos^{-1}\left(\frac{-1}{2}\right) = \frac{2\pi}{3}$

$C = \cos^{-1}\left(\frac{-1}{2}\right) = \frac{2\pi}{3}$

6. $\nabla f = \langle 3x^2 + yz, 3y^2 + xz, 3z^2 + xy \rangle = \langle 4, 4, 4 \rangle$

$u = \langle -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \rangle$

$\nabla f \cdot u = \langle 4, 4, 4 \rangle \cdot \langle -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \rangle = -\frac{4}{\sqrt{3}} - \frac{4}{\sqrt{3}} - \frac{4}{\sqrt{3}} = -\frac{12}{\sqrt{3}}$

7. $\frac{dy}{du} = \frac{dy}{dx} \cdot \frac{dx}{du} + \frac{dy}{dy} \cdot \frac{dy}{du} = bx \cdot e^u \cos v + (-by) \cdot e^u \cos v$

$= b(e^u \cos v) \cdot e^u \cos v + (-b(e^u \sin v)) \cdot e^u \cos v$

$= b(1 \cdot \cos 1) \cdot 1 \cdot \cos 1 + (-b \cdot \sin 1) \cdot \cos 1$

$= b \cos^2(1) - b \sin(1) \cos(1)$

- 8. $\text{div } F = 3 - 2 + 5 = 6$

$\int_0^2 \int_{-2}^0 \int_0^{\sqrt{4-x^2-y^2}} 6 \, dz \, dy \, dx = \int_0^2 \int_{-2}^0 6\sqrt{4-x^2-y^2} \, dy \, dx = \int_0^2 \left. \frac{-6y}{\sqrt{4-x^2-y^2}} \right|_{-2}^0 dx$

$= \int_0^2 \frac{12}{\sqrt{4-x^2}} dx = \text{Nonsense can't take square root of negative number, no real answers}$

9. $\int_0^1 \int_0^1 \int_0^{2x+3y} 1 \, dz \, dy \, dx = \int_0^1 \int_0^1 (2x+3y) \, dy \, dx = \int_0^1 \left. 2xy + \frac{3}{2}y^2 \right|_0^1 dx$

$\int_0^1 2x + \frac{3}{2} dx = x^2 + \frac{3}{2}x \Big|_0^1 = 1 + \frac{3}{2} = \frac{5}{2}$

- 10. $f_x = 4 - \frac{2}{2x+y}$
 $f_y = -2y - \frac{1}{2x+y}$
 $f_{xx} = \frac{4}{(2x+y)^2}$
 $f_{yy} = -2 + \frac{1}{(2x+y)^2}$
 $f_{xy} = \frac{2}{(2x+y)^2}$

$4 - \frac{2}{2x+y} = -2y - \frac{1}{2x+y}$

$4 + 2y = \frac{1}{2x+y}$

$2 = 2$
 $x = 3/4 \quad y = -1$

$(3/4, -1):$
 $D = f_{xx}f_{yy} - (f_{xy})^2$

$= (16)(2) - (8)^2$
 $= 32 - 64 = -32$

$(3/4, -1)$ is a saddle point
 b/c $D < 0$, according to the 2nd Derivative Test

$$11. \quad f_x = \frac{2x}{\sqrt{2x^2 + 3y^2 + z^2}} = \frac{2}{3} \quad f_y = \frac{3y}{\sqrt{2x^2 + 3y^2 + z^2}} = \frac{3}{3} = 1 \quad f_z = \frac{z}{\sqrt{2x^2 + 3y^2 + z^2}} = \frac{2}{3}$$

$$\frac{2}{3}(x-1) + 1(y-1) + \frac{2}{3}(z-2) = 0 \quad \frac{2}{3}x + y + \frac{2}{3}z = 3 \quad |(\cdot) 3|$$

$$\frac{2}{3}x + y + \frac{2}{3}z = -\frac{2}{3} - \frac{3}{3} - \frac{4}{3} = 0 \quad \frac{2}{3}(1.001) + 0.999 + \frac{2}{3}(-2.001) = 3.0003333$$

$$12. \quad \int_0^{\pi/4} \int_0^1 r \cos \theta \, r \, dr \, d\theta + \int_{\pi/4}^{\pi/2} \int_0^1 r \cos \theta \, r \, dr \, d\theta$$

$$\int_0^{\pi/4} \frac{r^3 \cos \theta}{3} \Big|_0^1 d\theta = \int_0^{\pi/4} \frac{1}{3} \cos \theta \, d\theta = \frac{1}{3} \sin \theta \Big|_0^{\pi/4} = \frac{\sqrt{2}}{6}$$

$$\int_{\pi/4}^{\pi/2} \frac{r^3 \cos \theta}{3} \Big|_0^1 d\theta = \frac{1}{3} \sin \theta \Big|_{\pi/4}^{\pi/2} = \frac{1}{3} - \frac{\sqrt{2}}{6}$$

$$= \frac{\sqrt{2}}{6} + \left(\frac{1}{3} - \frac{\sqrt{2}}{6} \right) = \frac{1}{3}$$

$$13. \quad \int_0^2 \int_0^{\sqrt{4-z^2}} \int_{-\sqrt{4-z^2-y^2}}^0 x^2 y z \, dx \, dy \, dz$$

$$\int_0^2 \int_0^{\pi} \int_0^2 r^2 \sin^2 \theta \cos^2 \theta \, r \sin \theta \sin \theta \, r \cos \theta \, r^2 \sin \theta \, dr \, d\theta \, dz$$

$$\int_0^{2\pi} \int_0^{\pi} \int_0^2 r^6 \sin^4 \theta \cos^3 \theta \sin \theta \, dr \, d\theta \, d\theta$$

$$14. \quad r'(t) = \langle 0, 3 \cos t, -3 \sin t \rangle$$

$$r''(t) = \langle 0, -3 \sin t, -3 \cos t \rangle \times \langle -9, 0, 0 \rangle$$

$$k(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3} = \frac{|\langle -9, 0, 0 \rangle|}{|\langle 0, 3 \cos t, -3 \sin t \rangle|^3} = \frac{9}{3^3} = \frac{9}{27} = \frac{1}{3}$$

$$15. \quad r_u = \langle 2u, v, 0 \rangle \times \langle 2v^2, -4uv, 2u^2 \rangle$$

$$r_v = \langle 0, u, 2v \rangle$$