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SSC: (II)

1. 18
2. $\int_0^1 \int_{y^2}^1 f(x,y)\,dx\,dy$
3. there's no z for this question.
4. $(-1, -8, 5)$
5. A: 60° B: 60° C: 60° Radians are all $\frac{\pi}{3}$
6. $-4\sqrt{3}$
7. 5.7869
8. 0
9. -15.
10. $\frac{3}{8}$. has a saddle point $(\frac{3}{4}, -1)$
11. $\frac{9001}{3000}$, 3.00033
12.
13. $\int_0^{\pi} \int_0^{2\pi} \int_0^2 \rho^5 \sin^6\phi \cos\phi \cos^2\theta \sin\theta \, d\rho\, d\theta\, d\phi$
14. $\frac{1}{3}$
15. $\int_0^1 \int_0^v u^2 + uv + v^2 \, du\, dv$
16. 14
17. DNE

Signed: Wenhao Li

1. $P = \cos(e^{\sin x}) + 5y \quad Q = \sin(e^{\cos y}) + 11x$

$\frac{dP}{dy} = 5 \quad \frac{dQ}{dx} = 11$

$\frac{dQ}{dx} - \frac{dP}{dy} = 11 - 5 = 6$

6. $\text{Area} = 6 \cdot (2\times1 + 2\times1\times\frac{1}{2})$

$= 6(2+1)$

$= 6\times3$

$= 18$

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2 ~~$0 \le x \le \sqrt{y}$~~ $0 \le y \le \sqrt{x}$

~~$\frac{1}{4} \le y \le 1$~~ $\frac{1}{4} \le x \le 1$

$y = \sqrt{x}$ $x = y^2$

$0 \le y \le 1$

$y^2 \le x \le 1$ $\int_0^1 \int_{y^2}^1 f(x,y)\,dx\,dy$.

3. $f_x = -2\sin(x+y)$ ~~$+$~~ $-4\sin(x+z)z' - 8\sin(y+z)z' = 0$

$f_y = -2\sin(x+y) - 4\sin(x+z)z' - 8\sin(y+z)z' = 0$

$f_z = -4\sin(x+z)z' - 8\sin(y+z)z' = 0$

$\frac{dz}{dx} = \frac{2\sin(x+y)}{-4\sin(x+z) - 8\sin(y+z)}$ $\frac{dz}{dy} = \frac{2\sin(x+y)}{-4\sin(x+z) - 8\sin(y+z)}$ $\frac{dz}{z} = 0$

plugin $(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6})$

$\frac{dz}{x} = -\frac{1}{6}$ $\frac{dz}{y} = -\frac{1}{6}$ $\frac{dz}{z} = 0$

$-\frac{1}{6}(x - \frac{\pi}{6}) - \frac{1}{6}(y - \frac{\pi}{6}) + 0 = 0$

$-\frac{1}{6}x + \frac{\pi}{36} - \frac{1}{6}y + \frac{\pi}{36} = 0$

$-\frac{1}{6}x - \frac{1}{6}y + \frac{\pi}{18} = 0$

4.

i	j	k
a	b	c
2a	-b	3c

$i(3bc + bc) - j(3ac - 2ac) + k(-ab - 2ab)$

$= 4bc\,i - ac\,j$ ~~$+$~~ $-3ab\,k$

$a \times b = \langle 1, 1, -1 \rangle$ $b \times c = \langle 1, -1, 1 \rangle$ $a \times c = \langle 2, 1, 2 \rangle$

$4\langle 1, -1, 1 \rangle - \langle 2, 1, 2 \rangle - 3\langle 1, 1, -1 \rangle$

$= \langle 4, -4, 4 \rangle - \langle 2, 1, 2 \rangle - \langle 3, 3, -3 \rangle$

$= \langle -1, -8, 5 \rangle$ $(a+b+c) \times (2a - b + 3c) = \langle -1, -8, 5 \rangle$

5. 又.

$AB = \langle 1,0,1\rangle = \sqrt{1^2+0^2+1^2} = \sqrt{2}$

$BC = \langle 0,1,-1\rangle = \sqrt{0+1^2+(-1)^2} = \sqrt{2}$

$CA = \langle -1,-1,0\rangle = \sqrt{(-1)^2+(-1)^2+0} = \sqrt{2}$

$AB \cdot BC = \langle 1,0,1\rangle \cdot \langle 0,1,-1\rangle$

$= 0+0-1 = -1$

$BC \cdot CA = \langle 0,1,-1\rangle \cdot \langle -1,-1,0\rangle$

$= 0-1+0 = -1$

$AB \cdot CA = \langle 1,0,1\rangle \cdot \langle -1,-1,0\rangle$

$= -1+0+0 = -1$

since the three dot products is the same.
the three angles are the same.

$180 \div 3 = 60$

angle A is $60°$ radians $= \frac{\pi}{3}$

angle B is $60°$ radians $= \frac{\pi}{3}$

angle C is $60°$ radians $= \frac{\pi}{3}$

6. $f_x = 3x^2 + yz$ $f_y = 3y^2 + xz$ $f_z = 3z^2 + xy$

$\nabla f \langle 3x^2+yz,\ 3y^2+xz,\ 3z^2+xy\rangle$

$|\langle -1,-1,-1\rangle| = \sqrt{(-1)^2+(-1)^2+(-1)^2} = \sqrt{3}$

$u = \frac{1}{\sqrt{3}}\langle -1,-1,-1\rangle = \langle -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\rangle$

$\nabla f(1,1,1) = \langle 3+1, 3+1, 3+1\rangle = \langle 4,4,4\rangle$

$\langle -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\rangle \cdot \langle 4,4,4\rangle$

$= -\frac{1}{\sqrt{3}} \times 4 - \frac{1}{\sqrt{3}} \times 4 - \frac{1}{\sqrt{3}} \times 4$

$= -\frac{4}{\sqrt{3}} - \frac{4}{\sqrt{3}} - \frac{4}{\sqrt{3}}$

$= -4\sqrt{3}$

7. $\frac{dg}{du} = \frac{dg}{dx}\cdot\frac{dx}{du} + \frac{dg}{dy}\cdot\frac{dy}{du}$

$= 6x\cdot(e^u\cos v + e^u(-\sin v)) - 6y(e^u\sin v + e^u\cos v)$

$x = e^u\cos v$ $y = e^u\sin v$

plug in $(u, v) = (0, 1)$

$x = e^0\cos 1 = \cos 1$ $y = e^0\sin 1 = \sin 1$

$\frac{dg}{du} = 6\cos 1(e^0\cos 1 + e^0(-\sin 1)) - 6\sin 1(e^0\sin 1 + e^0\cos 1)$

$= 5.7869$

8. For this problem surface integral

Because it is a closed surface

So for the surface integral over any closed surface,

the answer have to be zero.

So $\int_S F\cdot ds = 0$.

9. $g = 2x + 3y$

$P = 3z$ $Q = 2x$ $R = y + z$

$\iint_D (-3z(2) - 2x(3) + y + z)\,dA$

$= \iint_D (-6z - 6x + y + z)\,dA$

$= \iint_D (-6x + y - 5z)\,dA$

since $z = 2x + 3y$

$\iint_D (-6x + y - 5(2x + 3y))\,dA$

$\iint_D (-6x + y - 10x - 15y)\,dA$

$= \iint_D (-16x - 14y)\,dA$

$0 < x < 1$ $0 < y < 1$

$\int_0^1\int_0^1 (-16x - 14y)\,dx\,dy$

$\int_0^1 -16x - 14y\,dx$

$= -16\frac{x^2}{2} - 14xy\Big|_0^1$

$= -8x^2 - 14xy\Big|_0^1 = -8 - 14y$

$\int_0^1 -8 - 14y\,dy$

$= -8y - \frac{14y^2}{2}\Big|_0^1$

$= -8y - 7y^2\Big|_0^1 = -8 - 7 = -15$

10.

$f_x = 4 - \frac{1}{2x+y} \cdot 2 = 4 - \frac{2}{2x+y}$

$f_y = -2y - \frac{1}{2x+y}$

$f_{xx} = \frac{4}{(2x+y)^2}$

$f_{xy} = \frac{-4(4x+2y)}{(2x+y)^4}$

$f_{yy} = -2 + \frac{1}{(2x+y)^2}$

$4 - \frac{2}{2x+y} = 0 \qquad -2y - \frac{1}{2x+y} = 0$

~~$-2y - \frac{1}{2x+y}$~~ $-2y = \frac{1}{2x+y}$

$\frac{2}{2x+y} = -4y$

$4 + 4y = 0$

$y = -1 \quad x = \frac{3}{4}$

$(\frac{3}{4}, -1)$

$f_{xx}(\frac{3}{4}, -1) = \frac{4}{(2 \times \frac{3}{4} - 1)^2} = 16$

$f_{xy}(\frac{3}{4}, -1) = \frac{-4(4 \times \frac{3}{4} - 2)}{(2 \times \frac{3}{4} - 1)^4} = \frac{-4}{\frac{1}{16}} = -64$

$f_{yy} = -2 + \frac{1}{(2 \times \frac{3}{4} - 1)^2} = 2$

$D = 16 \times 2 - (-64)^2 = -4064 < 0$

So. $(\frac{3}{4}, -1)$ is a saddle point

11. $f = (2x^2 + 3y^2 + z^2)^{\frac{1}{2}}$

$f_x = \frac{1}{2}(2x^2 + 3y^2 + z^2)^{-\frac{1}{2}} \cdot 6x$

$f_y = \frac{1}{2}(2x^2 + 3y^2 + z^2)^{-\frac{1}{2}} \cdot 6y$

$f_z = \frac{1}{2}(2x^2 + 3y^2 + z^2)^{-\frac{1}{2}} \cdot 2z$

plug in (1, 1, 2)

$f_x = \frac{2}{3} \qquad f_y = 1 \qquad f_z = \frac{2}{3}$

$f(1,1,2) = 3$

$L(x,y,z) = 3 + \frac{2}{3}(x-1) + (y-1) + \frac{2}{3}(z-2)$

$= 3 + \frac{2}{3}(1.001 - 1) + (0.999 - 1) + \frac{2}{3}(2.001 - 2) = 3.00033 = \frac{9001}{3000}$

12.

$$\int_0 \int_0^{2\sec\theta} r^2 \sin\theta \, dr \, d\theta$$

13. ~~$-\sqrt{4-z^2}$~~ $\le x \le 0$

$0 \le y \le \sqrt{4-z^2}$

$0 \le z \le 2$

~~$0 \le r \le 2$~~ $0 \le \theta \le 2\pi$ $0 \le \phi \le \pi$

$0 \le \rho \le 2$

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$

$dx\,dy\,dz = \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

~~$\int_0^2 \int_0^{2\pi} \int_0^{\pi}$~~ $(\rho \sin\phi \cos\theta)^2 \cdot \rho \sin\phi \sin\theta \cdot \rho\cos\phi \cdot \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

~~$\int_0^2 \int_0^{2\pi} \int_0^{\pi}$~~ $\int_0^{\pi} \int_0^{2\pi} \int_0^2 \rho^5 \sin^4\phi \cos\phi \cos^2\theta \sin\theta \, d\rho \, d\theta \, d\phi$

14.

$r'(t) = \langle 0, 3\cos t, -3\sin t \rangle$

$r''(t) = \langle 0, -3\sin t, -3\cos t \rangle$

$t = \frac{\pi}{3}$

plug in $t = \frac{\pi}{3}$

$r'(t) = \langle 0, \frac{3}{2}, -\frac{3\sqrt{3}}{2} \rangle$

$r''(t) = \langle 0, -\frac{3\sqrt{3}}{2}, -\frac{3}{2} \rangle$

$r'(t) \times r''(t)$

$$\begin{matrix} i & j & k \\ 0 & \frac{3}{2} & -\frac{3\sqrt{3}}{2} \\ 0 & -\frac{3\sqrt{3}}{2} & -\frac{3}{2} \end{matrix}$$

$i(\frac{3}{2} \times (-\frac{3}{2})) - \frac{9 \times 3}{4}) - j(0-0) + k(0-0)$

$= (-\frac{9}{4} - \frac{27}{4})i - 0j + 0k$

$= -9i - 0j + 0k = \langle -9, 0, 0 \rangle$

$|r'(t) \times r''(t)| = \sqrt{81 + 0 + 0} = 9$

$|r'(t)| = \sqrt{0 + \frac{9}{4} + \frac{9 \times 3}{4}} = 3$

$k(t) = \frac{9}{3^3} = \frac{9}{27} = \frac{1}{3}$

15.

$0<u<v<1$

$0<u<v$

$0<v<1$

$\int_0^1 \int_0^v u^2+uv+v^2 \, du \, dv$.

16. $f_x = y^2 z^3$ $f_y = 2yxz^3$ $f_z = 3z^2 xy^2$

$\nabla f = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$

plug in $(1,1,1)$

$\nabla f = \langle 1, 2, 3 \rangle$

$g_x = 1$ $g_y = 2y$ $g_z = 3z^2$

$\nabla g = \langle 1, 2y, 3z^2 \rangle$

plug in $(1,1,1)$

$\nabla g = \langle 1, 2, 3 \rangle$

$\nabla f \cdot \nabla g = \langle 1,2,3 \rangle \cdot \langle 1,2,3 \rangle$

$= 1+4+9$

$= 14$

17. $$\lim_{(x,y,z,w)\to(0,0,0,0)} \frac{(x+y)^2-(z+w)^2}{x+y-z-w}$$

$$\frac{(0+0)^2-(0+0)^2}{0+0-0-0} = \frac{0}{0}$$

$y=cx \quad z=dx \quad w=ax$

$$\frac{(x+cx)^2-(dx+ax)^2}{x+cx-dx-ax}$$

$$= \frac{x^2+2xcx+(cx)^2-((dx)^2+2dxax+(ax)^2)}{x(1+c-d-a)}$$

$$= \frac{x(x+2cx+c^2x-d^2x-2dax-a^2x)}{x(1+c-d-a)}$$

$$= \frac{x+2cx-c^2x-d^2x-2dax-a^2x}{1+c-d-a}$$

Since this depends on the slope c, d, a, we get different limits for different lines, so the limit does not exist.

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