Name: SHUBIN XIE RUID: 203002353.

SSC: I and II.

MATH 251 (22, 23, 24) Fall 2020, Final exam, Tue, Dec. 15, 2020.

my exam time (Wed, Dec 16, 2020 (china time)).

Email the completed test, renamed as final first last. pdf. to Drzcalc3@gmail.com no later than 3:30 PM (or in case of conflict, three and half hour after the start).

my reply time (12:30 PM for china) (EST 11:30 PM).

write your final answers below:

1. -18.

2. $\int_0^{\frac{1}{2}}\int_{\frac{1}{4}}^{1} f(x,y)\,dx\,dy + \int_{\frac{1}{2}}^{1}\int_{y^2}^{1} f(x,y)\,dx\,dy$.

3. $z = -\frac{1}{2}x - \frac{5}{9}y + \frac{7}{18}\pi$.

4. $\langle -1, -8, 5\rangle$

5. $A: \frac{\pi}{3}$ $B: \frac{\pi}{3}$ $C: \frac{\pi}{3}$.

6. $-\frac{24}{\sqrt{12}}$.

7. $6\cos^2 1 - 6\sin 1\cos 1$.

8. 8π.

9. $-\frac{33}{2}$.

10. $(\frac{3}{4}, -1)$ is ~~unknow point~~ local min point.

11. 3.000333333.

12. $\frac{1}{3}$.

13. $\cancel{\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\int_{\frac{\pi}{2}}^{\pi}\int_0^2 (\rho}$ $\int_0^{\frac{\pi}{2}}\int_{\frac{\pi}{2}}^{\pi}\int_0^2 (\rho\sin\phi\cos\theta)^2\cdot(\rho\sin\phi\,\cancel{\cos}\,\sin\theta)\cdot(\rho\cos\phi)\cdot\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$.

14. $\frac{1}{3}$.

15. $\int_0^1\int_0^1 \mathrm{curl}\,(F(u,v))\cdot(0,g,k)\,du\,dv$.

16. 14.

17. limit exist and it is 0.

Sign the following declaration:

I (SHUBIN XIE) hereby declare that all the work was done by myself. I was allowed to use Maple (unless specifically told not to), calculators, the book, and all the material in the web-page of this class but not other resources on the internet.

I only spent (at most) 3 hours on doing the exam. The last 30 minutes were spent in checking and double-checking the answers.

I also understand that I may be subject to a random short chat to verify that I actually did it all by my self.

Signed: SHUBIN XIE.

1, without using maple (or any software) compute the vector-field line integral.

$$\int_C (\cos(e^{\sin x})+5y)\,dx+(\sin(e^{\cos y})+11x)\,dy$$

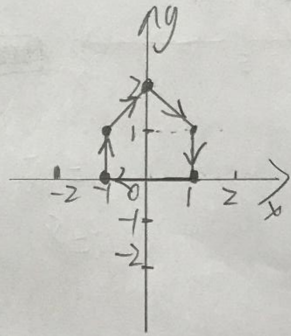
over the path consisting of the five line segments (in that order)

$$(1,0)\to(-1,0)\to(-1,1)\to(0,2)\to(1,1)\to(1,0).$$

Explain!

Ans:

Use green theorem.



$$\iint\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dA$$

$$\frac{\partial Q}{\partial x}=11 \qquad \frac{\partial P}{\partial y}=5.$$

$$\iint(11-5)\,dA=\iint 6\,dA$$

Because is clockwise so mupltipule $\cdot(-1)$

$$-\iint 6\,dA=-6\iint dA=-6\cdot \text{area}=-6\cdot[2\times1+\cancel{2}^{1}\times1\times\tfrac{1}{\cancel{2}}]$$

$$=-6\cdot[2+1]$$

$$=-6\cdot 3$$

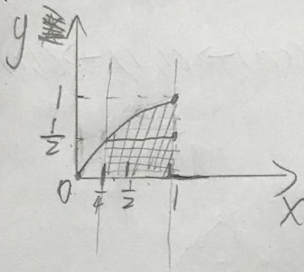
$$=-18.$$

Ans: -18.

7. ~~comput~~ change the order of integration.

$$\int_{\frac{1}{4}}^{1} \int_{0}^{\sqrt{x}} f(x,y)\,dy\,dx.$$

Ans:



$$\int_{0}^{1} \int_{y^2+\frac{1}{4}}^{\frac{5}{4}} f(x,y)\,dx\,dy$$

~~final ans~~ $\int_{0}^{\frac{1}{2}} \int_{\frac{1}{4}}^{1} f(x,y)\,dx\,dy + \int_{\frac{1}{2}}^{1} \int_{y^2}^{1} f(x,y)\,dx\,dy$

$y = \sqrt{x}$

$y^2 = x$

$\boxed{\text{final ans: } \int_{0}^{\frac{1}{2}} \int_{\frac{1}{4}}^{1} f(x,y)\,dx\,dy + \int_{\frac{1}{2}}^{1} \int_{y^2}^{1} f(x,y)\,dx\,dy}$

3、Find the equation of the tangent plane at the point $\left(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}\right)$ to the surface given implicitly by

$$2\cos(x+y) + 4\cos(x+z) + 8\cos(y+z) = 7.$$

Express you answer in explicit form, i.e in the format $z = ax + by + c$

Ans: $z=$

~~Ans:~~

$$\text{grad} f = \langle -2\sin(x+y) - 4\sin(x+z),\ -2\sin(x+y) - 8\sin(y+z),\ -4\sin(x+z) - 8\sin(y+z) \rangle$$

$$= \langle \sin(x+y) + 2\sin(x+z),\ \sin(x+y) + 4\sin(y+z),\ 2\sin(x+z) + 4\sin(y+z) \rangle.$$

plug $\left(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}\right)$

$$\text{grad} f = \langle \sin\frac{\pi}{3} + 2\sin\frac{\pi}{3},\ \sin\frac{\pi}{3} + 4\sin\frac{\pi}{3},\ 2\sin\frac{\pi}{3} + 4\sin\frac{\pi}{3} \rangle$$

$$= \langle \frac{\sqrt{3}}{2} + \overset{1}{\cancel{2}} \cdot \frac{\sqrt{3}}{\cancel{2}_{1}},\ \frac{\sqrt{3}}{2} + \overset{2}{\cancel{4}} \cdot \frac{\sqrt{3}}{\cancel{2}_{1}},\ \overset{1}{\cancel{2}} \cdot \frac{\sqrt{3}}{\cancel{2}_{1}} + \overset{2}{\cancel{4}} \cdot \frac{\sqrt{3}}{\cancel{2}_{1}} \rangle$$

$$= \langle \frac{3}{2}\sqrt{3},\ \frac{5}{2}\cancel{\sqrt{3}},\ 3\cancel{\sqrt{3}} \rangle$$

$$= \langle 3,\ 5,\ 6 \rangle$$

equation:

$$3\left(x - \frac{\pi}{6}\right) + 5\left(y - \frac{\pi}{6}\right) + 6\left(\cancel{y}\, z - \frac{\pi}{6}\right) = 0.$$

$$3x - \frac{\pi}{2} + 5y - \frac{5}{6}\pi + 6z - \pi = 0.$$

$$3x + 5y + 6z - \frac{7}{3}\pi = 0.$$

$$6z = -3x - 5y + \frac{7}{3}\pi$$

$$z = -\frac{1}{2}x - \frac{5}{6}y + \frac{7}{18}\pi$$

Ans: $z = -\frac{1}{2}x - \frac{5}{6}y + \frac{7}{18}\pi$.

7.4 Let a, b, c be three vectors such that

$a\times b = i+j-k$, $b\times c = i-j+k$, $a\times c = 2i+j+2k$

what is

$(a+b+c) \times (2a-b+3c)$?

Ans:

$$\begin{vmatrix} i & j & k \\ a & b & c \\ 2a & -b & 3c \end{vmatrix}$$

$i(b\cdot 3c-(-b\cdot c))-j(a\cdot 3c-2a\cdot c)+k(-ab-2ab)$

$=i(b\cdot 3c+b\cdot c)-j(3ac-2ac)+k(-3ab)$

$=\langle 4bc, -ac, -3ab\rangle$

$bc=\langle 1,-1,1\rangle$ $4bc=\langle 4,-4,4\rangle$

$ac=\langle 2,1,2\rangle$ $-ac=\langle -2,-1,-2\rangle$

$ab=\langle 1,1,-1\rangle$ $-3ab=\langle -3,-3,3\rangle$

$(a+b+c)\times(2a-b+3c)$

$=\langle -1,-8,5\rangle$

Ans: $\langle -1,-8,5\rangle$.

5. Find the three angles of the triangle ABC where

$A = (0,0,0)$, $B = (1,0,1)$, $C = (1,1,0)$

Ans: The angle at A is: $\frac{\pi}{3}$ radians.

The angle at B is: $\frac{\pi}{3}$ radians

The angle at C is: $\frac{\pi}{3}$ radians.

~~AB: $\cos\theta = \frac{0}{(|A||B|)} = 0$~~

Ans: $AB = \langle 1, 0, 1 \rangle$

$AC = \langle 1, 1, 0 \rangle$

$BC = \langle 0, 1, -1 \rangle$

$BA = \langle -1, 0, -1 \rangle$

$CB = \langle 0, -1, 1 \rangle$

$CA = \langle -1, -1, 0 \rangle$

$A: \cos\theta = \frac{1+0+0}{(\sqrt{2} \cdot \sqrt{2})} = \frac{1}{2}$

$\theta = 60^\circ = \frac{\pi}{3} = \frac{\pi}{3}$

$B = \cos\theta = \frac{1}{(\sqrt{2} \cdot \sqrt{2})} = \frac{1}{2}$

$\theta = 60^\circ = \frac{\pi}{3}$

$C = \cos\theta = \frac{1}{(\sqrt{2} \cdot \sqrt{2})} = \frac{1}{2}$

$\theta = 60^\circ = \frac{\pi}{3}$

6、Find the directional derivative of

$$f(x,y,z) = x^3+y^3+z^3+\cancel{}xyz,$$

at the point(1,1,1) in a direction pointing to the point(-1,-1,-1)

Ans: direction (-1,-1,-1) - (1,1,1) = $\langle -2,-2,-2 \rangle$

grad $f = \langle 3x^2+yz,\ 3y^2+xz,\ 3z^2+xy \rangle$.

plug (1,1,1) in:

~~grad~~ grad $f = \langle 3+1,\ 3+1,\ 3+1 \rangle$

$= \langle 4,\ 4,\ 4 \rangle$.

unit direction: $\dfrac{\langle -2,-2,-2 \rangle}{\sqrt{4+4+4}} = \dfrac{\langle -2,-2,-2 \rangle}{\sqrt{12}} = \cancel{} \left\langle -\dfrac{2}{\sqrt{12}}, -\dfrac{2}{\sqrt{12}}, -\dfrac{2}{\sqrt{12}} \right\rangle$

unit direction · grad f

$= \cancel{\left\langle \frac{2}{\sqrt{12}} \right.} \left\langle -\dfrac{2}{\sqrt{12}}, -\dfrac{2}{\sqrt{12}}, -\dfrac{2}{\sqrt{12}} \right\rangle \cdot \langle 4,4,4 \rangle$

$= -\dfrac{8}{\sqrt{12}} - \dfrac{8}{\sqrt{12}} - \dfrac{8}{\sqrt{12}}$

$= -\dfrac{24}{\sqrt{12}}$

Ans: $-\dfrac{24}{\sqrt{12}}$.

because I am not sure it can cancle out or not

Therefore ~~so~~ I am not cancle out with each other.

☆. Using the chain rule (no credit for other methods), find

$$\frac{\partial g}{\partial u}.$$

at $(u,v)=(0,1)$, where

$$g(x,y)=3x^2-3y^2$$

and

$$x=e^u\cos v \quad y=e^u\sin v$$

Ans: $\frac{\partial g}{\partial u}=\frac{\partial g}{\partial x}\cdot\frac{\partial x}{\partial u}+\frac{\partial g}{\partial y}\cdot\frac{\partial y}{\partial u}$.

$\frac{\partial g}{\partial x}=6x \quad \frac{\partial g}{\partial y}=-6y$

$\frac{\partial x}{\partial u}=\cos v\, e^u \quad \frac{\partial y}{\partial u}=e^u\cos v$

plug $(u,v)=(0,1)$ in.

$\frac{\partial x}{\partial u}=\cos 1\, e^0=\cos 1 \quad \frac{\partial y}{\partial u}=e^0\cos 1=\cos 1$

$x=e^0\cos 1=\cos 1 \quad y=e^0\sin 1=\sin 1$

$\frac{\partial g}{\partial u}=6\cdot\cos 1\cdot\cos 1+(-6\cdot\sin 1)\cdot\cos 1$

$=6\cos^2 1-6\sin 1\cos 1$

Ans: $6\cos^2 1-6\sin 1\cos 1$.

8, without using maple (or any other software), compute the vector field surface integral $\int_S F \cdot ds$ if

$$F = \langle 3x + \cos(y^3 + yz),\ -2y + e^{x+z^2},\ 5z + \sin(xy^3 + e^x) \rangle$$

and S is the closed surface in 3D space bounding the region.

$$\{(x,y,z):\ x^2+y^2+z^2<4 \text{ and } x>0 \text{ and } y<0 \text{ and } z>0\}$$

Ans:

because calculate $\int_S F \cdot ds$ and it is closed surface

so we use divergence theorem.

$$\operatorname{div} F = 3 + (-2) + 5 = 1+5 = 6.$$

$$\int_S F \cdot ds = \iiint_R 6\, dx\, dy\, dz$$

$$= 6 \cdot \text{volume}$$

$$= 6 \cdot \frac{4}{3}\pi r^3 \cdot \frac{1}{8}$$

$$= 6 \cdot \frac{4}{3}\pi 2^3 \cdot \frac{1}{8}$$

$$= \cancel{6}^{2} \cdot \frac{4}{\cancel{3}_{1}}\pi \cdot \cancel{8}^{1} \cdot \frac{1}{\cancel{8}_{1}}$$

$$= 8\pi.$$

Ans: 8π

Q. Compute the vector-field surface integral $\iint_S F \cdot dS$ if

$$F = \langle 3z, 3x, y+z \rangle,$$

and S is the oriented surface.

$$z = 2x+3y, \quad 0<x<1, \quad 0<y<1.$$

with upward pointing normal.

Ans:

$$\iint_S F \cdot dS = \iint_D \left(-P\frac{dg}{dx} - Q\frac{dg}{dy} + R\right) dA.$$

Because upward, we don't need to multiple (-1).

$P = 3(2x+3y) = 6x+9y$ $\qquad \frac{dg}{dx} = 2.$

$\qquad\qquad\qquad\qquad\qquad \frac{dg}{dy} = 3.$

$Q = 3x$

$R = y + 2x+3y = 2x+4y.$

$$\int_0^1\int_0^1 \left(-[(6x+9y)\cdot 2] - [3x\cdot 3] + [2x+4y]\right) dx\,dy$$

$$= \int_0^1\int_0^1 [-12x - 18y - 9x + 2x + 4y]\,dx\,dy$$

$$= \int_0^1\int_0^1 [-19x - 14y]\,dx\,dy$$

$$= -\frac{33}{2}$$

Ans: $-\frac{33}{2}$

10, without using maple or software, find the critical point(s) of

$$f(x,y) = 4x - y^2 - \ln(2x+y),$$

and decide for each whether it a local maximum, local minimum, or saddle point. Explain.

Ans:

$f\_x = 4 - \frac{2}{2x+y}$ $\qquad f\_xx = \frac{4}{(2x+y)^2} = \frac{4}{\cancel{\frac{3}{2}-1}(\frac{1}{2})^2} = 16$

$f\_xy = \frac{2}{(2x+y)^2} = \cancel{2}\ 8$

$f\_y = -2y - \frac{1}{2x+y}.$ $\qquad f\_yy = 2 + \frac{1}{(2x+y)^2} = \cancel{5}\ 2.$

$4 - \frac{2}{2x+y} = 0$

$\frac{\cancel{2}\,1}{2x+y} = \cancel{4}\,2$

$\cancel{2}\,1 = 4x + 2y$

$2y + 4x = 1$

critical point $(\frac{3}{4}, -1)$

$-2y - \frac{1}{2x+y} = 0$

$-2y = \frac{1}{2x+y}$

$-4xy - 2y^2 = 1$

$4xy + 2y^2 = -1$

$y(4x+2y) = -1$

$4x + 2y = 1$

$y = -1$

$4x = 1 + 2$

$4x = 3$

$x = \frac{3}{4}.$

$D = \cancel{f\_xx}\ 16 \times \cancel{3}\,2 - 8^2$

$= \cancel{64 - 64}\ 32 - 64$

$= \cancel{0}\ -32 < 0.$

Because $\cancel{D=0}$ $D < 0$ $\qquad f\_xx = 16 > 0$

~~point $(\frac{3}{4}, -1)$ is unknow~~ ~~unknow~~

point $(\frac{3}{4}, -1)$ is local min.

1) without using maple or software, using a linearization around the point (1,1,2), approximate f(1.001, 0.999, 2.001) if

$$f(x,y,z)=\sqrt{2x^2+3y^2+z^2}$$

Ans:

$$f(a,b,c)+f_x(a,b,c)(x-a)+f_y(a,b,c)(y-b)+f_z(a,b,c)(z-c).$$

$L(x,y) = \cancel{f(a,b)+f_x(a}$

$$f_x(a,b,c)=\frac{4x}{2\sqrt{2x^2+3y^2+z^2}}=\frac{2x}{\sqrt{2x^2+3y^2+z^2}}=\frac{2}{3}$$

$\cancel{u=2x^2+3y^2+z^2}$

$\cancel{du=4x\,dx}$

$\cancel{=dx}$

$$f_y(a,b,c)=\frac{6y}{2\sqrt{2x^2+3y^2+z^2}}=\frac{3y}{\sqrt{2x^2+3y^2+z^2}}=\frac{3}{3}=1$$

$$f_z(a,b,c)=\frac{2z}{2\sqrt{2x^2+3y^2+z^2}}=\frac{z}{\sqrt{2x^2+3y^2+z^2}}=\frac{2}{3}$$

$$f(a,b,c)=\sqrt{2\cdot1^2+3\cdot1^2+2^2}=\sqrt{2+3+4}=\sqrt{9}=3.$$

$$L(x,y)=3+\frac{2}{3}(x-1)+(y-1)+\frac{2}{3}(z-2)$$

$$L(1.001,0.999,2.001)=3+\frac{2}{3}(1.001-1)+(0.999-1)+\frac{2}{3}(2.001-2)$$

$$=3+\frac{2}{3}\cdot0.001-0.001+\frac{2}{3}\cdot0.001$$

$$=3.000333333.$$

12, without using maple (or any other software) and by using polar coordinates (no credit for doing it directly) find.

$$\int_0^{\frac{\sqrt{2}}{2}} \int_0^{x} x\,dy\,dx + \int_{\frac{\sqrt{2}}{2}}^{1} \int_0^{\sqrt{1-x^2}} x\,dy\,dx$$

Explain!:

Ans:

$$\int_0^{\frac{\sqrt{2}}{2}} \int_0^{x} x\,dy\,dx = \int_0^{\frac{\sqrt{2}}{2}} x^2\,dx = \frac{1}{3}x^3 \Big|_0^{\frac{\sqrt{2}}{2}}$$

$$= \frac{1}{3}\cdot\left(\frac{\sqrt{2}}{2}\right)^3$$

$$= \frac{1}{3}\cdot\frac{\cancel{2}\sqrt{2}}{\cancel{8}\,4}$$

$$= \frac{\sqrt{2}}{12}.$$

$$\int_{\frac{\sqrt{2}}{2}}^{1} \int_0^{\sqrt{1-x^2}} x\,dy\,dx$$

$dy\,dx = r\,dr\,d\theta$

$x = r\cos\theta$

$\{(r,\theta) \mid \frac{\sqrt{2}}{2} \le r \le 1,\ 0 \le \theta \le \frac{\pi}{2}\}.$

$$\int_0^{\frac{\pi}{2}} \int_{\frac{\sqrt{2}}{2}}^{1} r\cos\theta \cdot r\,dr\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_{\frac{\sqrt{2}}{2}}^{1} r^2\cos\theta\,dr\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{3}r^3\cos\theta \Big|_{\frac{\sqrt{2}}{2}}^{1} d\theta$$

$$= \frac{1}{3}\int_0^{\frac{\pi}{2}} \cos\theta \left(1 - \frac{\cancel{2}\sqrt{2}}{\cancel{8}\,4}\right) d\theta$$

$$= \frac{1}{3}\int_0^{\frac{\pi}{2}} \cos\theta \left(\frac{4-\sqrt{2}}{4}\right) d\theta$$

$$= \frac{4-\sqrt{2}}{12}\int_0^{\frac{\pi}{2}} \cos\theta\,d\theta = \frac{4-\sqrt{2}}{12}\sin\theta\Big|_0^{\frac{\pi}{2}} = \frac{4-\sqrt{2}}{12}(1-0) = \frac{4-\sqrt{2}}{12}.$$

Ans: $\frac{4-\cancel{\sqrt{2}}}{12} + \frac{\cancel{\sqrt{2}}}{12}$

$= \frac{\cancel{4}\,1}{\cancel{12}\,3}$

$= \frac{1}{3}$

13. Convert the triple iterated integral

$$\int_0^{2}\int_0^{\sqrt{4-z^2}}\int_{-\sqrt{4-z^2-y^2}}^{0} x^2 yz \, dx\,dy\,dz$$

to spherical coordinates. Do not evaluate.

Ans:

$$\{(\rho,\theta,\phi) \mid \rho = 0..2,\quad \theta = \cancel{0..\tfrac{\pi}{2}}\ \cancel{0..\tfrac{\pi}{2}}\ \tfrac{\pi}{2}..\pi,\quad \phi = \cancel{\tfrac{\pi}{2}..\pi}\ 0..\tfrac{\pi}{2}\}$$

$$\int_0^{\frac{\pi}{2}}\int_{\frac{\pi}{2}}^{\pi}\int_0^{2} (\rho\sin\phi\cos\theta)^2 \cdot (\rho\sin\phi\,\cancel{\cos\theta}\,\sin\theta) \cdot (\rho\cos\phi) \cdot \rho^2\sin\phi \, d\rho\,d\theta\,d\phi.$$

final Ans:

$$\int_0^{\frac{\pi}{2}}\int_{\frac{\pi}{2}}^{\pi}\int_0^{2} (\rho\sin\phi\cos\theta)^2 \cdot (\rho\sin\phi\sin\theta) \cdot (\rho\cos\phi) \cdot \rho^2\sin\phi \, d\rho\,d\theta\,d\phi$$

14. Find the curvature of the curve

$$r(t) = \langle 5, 3\sin t, 3\cos t \rangle$$

at the point where $t = \frac{\pi}{3}$.

Ans:

$$k(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

$$r'(t) = \langle 0, 3\cos t, -3\sin t \rangle = \langle 0, 3\cos\frac{\pi}{3}, -3\sin\frac{\pi}{3} \rangle$$
$$= \langle 0, 3\cdot\frac{1}{2}, -3\cdot\frac{\sqrt{3}}{2} \rangle$$
$$= \langle 0, \frac{3}{2}, -\frac{3}{2}\sqrt{3} \rangle$$

$$r''(t) = \langle 0, -3\sin t, -3\cos t \rangle$$
$$= \langle 0, -3\sin\frac{\pi}{3}, -3\cos\frac{\pi}{3} \rangle$$
$$= \langle 0, -3\cdot\frac{\sqrt{3}}{2}, -3\cdot\frac{1}{2} \rangle$$
$$= \langle 0, -\frac{3}{2}\sqrt{3}, -\frac{3}{2} \rangle$$

$$k(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

$$\begin{matrix} i & j & k \\ 0 & \frac{3}{2} & -\frac{3}{2}\sqrt{3} \\ 0 & -\frac{3}{2}\sqrt{3} & -\frac{3}{2} \end{matrix}$$

$$= \frac{\langle -9, 0, 0 \rangle}{3^3} = \frac{9}{3^3} = \frac{3^2}{3^3} = \frac{1}{3}.$$

Ans: $\frac{1}{3}$

16. set-up an iterated double integral, in type I format, but do not compute, for the surface area of the surface given parametrically by

$r(u,v) = \langle u^2, uv, v^2 \rangle, \quad 0 < u < v < 1.$

~~x=1~~ $\quad 0 < x, y < 1 \quad z = 1$

Ans:

$\cancel{\iint_S F \cdot dS = \iint_D (-P \; dy}$

$\cancel{0 < u < v < 1} \quad \cancel{0 < x, v < 1}$

$\iint_S \text{curl} F \cdot dS$

$\int_0^1 \int_0^1 \text{curl}(r(u,v)) dS = \cancel{\int_0^1 \int_0^1 v^2 \, du \, dv}$

$\int_0^1 \int_0^1 \text{curl}(r(u,v)) \cdot (0,0,k) \, du \, dv$

$\cancel{\text{Ans: } \int_0^1 \int_0^1 v^2}$

Ans: $\int_0^1 \int_0^1 \text{curl}((r(u,v)) \cdot (0,0,k) \, du \, dv.$

16. Let

$$f(x,y,z)=xy^2z^3$$

and let

$$g(x,y,z)=x+y^2+z^3$$

compute the dot-product

$$\text{grad}(f)\cdot\text{grad}(g)$$

at the point $(1,1,1)$:

Ans:

$$\text{grad}(f)=\langle y^2z^3,\ 2xyz^3,\ 3xy^2z^2\rangle$$

$$=\langle 1,\ 2,\ 3\rangle$$

$$\text{grad}(g)=\langle 1,\ 2y,\ 3z^2\rangle$$

$$=\langle 1,\ 2,\ 3\rangle$$

$$\text{grad}(f)\cdot\text{grad}(g)$$

$$=1+4+9=14$$

Ans: 14

17. Decide whether the following limit exists. If it does, find it. If it does not, explain why it does not exist.

$$\lim_{(x,y,z,w)\to(0,0,0,0)} \frac{(x+y)^2-(z+w)^2}{x+y-z-w}$$

$(x,0,0,0)$: $\frac{x^2}{x} = x = 0$.

$(0,y,0,0)$ $\frac{y^2}{y} = y = 0$

$(0,0,z,0)$ $\frac{-z^2}{-z} = z = 0$

$(0,0,0,w)$ $\frac{-w^2}{-w} = w = 0$.

limit exist. is 0.