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SSC: (circle) None / I / II / I and II

MATH 251 (22,23,24) [Fall 2020], Dr. Z. , Final Exam , Tue., Dec. 15, 2020

Email the completed test, renamed as `finalFirstLast.pdf` to DrZcalc3@gmail.com no later than 3:30pm, (or, in case of conflict, three and half hours after the start).

WRITE YOUR FINAL ANSWERS BELOW

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1. -18
2. $\int_0^1 \int_{y^2}^1 F(x,y)\,dx\,dy - \int_0^{1/2} \int_{y^2}^{1/4} F(x,y)\,dx\,dy$
3. $z = -\frac{x}{2} - \frac{5}{6}y + \frac{14\pi}{36}$
4.
5. Angle A = $\pi/3$ radians; Angle B = $\pi/3$ radians; Angle C = $\pi/3$ Radians
6. $-4\sqrt{3}$
7. $6(\cos^2(1) - \sin^2(1))$
8. $\int_{3\pi/2}^{2\pi} \int_0^2 -(3x+\cos(y^3+y^2))\,dx + (-2y+e^{x+z^2})\,dy + (5z+\sin(xy^3+e^x))\,r\,dr\,d\theta$
9. -15
10. $(3/4, -1)$ is a saddle point
11. $2/3\, x + y + \frac{2}{3} z$
12. $\sqrt{2}/6$
13. $\int_0^{\pi/2} \int_{\pi/2}^{\pi} \int_0^{\sqrt{2}} \rho^6 \sin^4\phi \cos\phi \cos^2\theta \sin\theta \, d\rho\, d\theta\, d\phi$
14. $1/3$
15. $\int_0^1 \int_u^1 F(r(u,v)) \cdot \sqrt{4v^4 + 16v^2u^2 + 4u^4}\, dv\, du$
16. 14
17. 0

Sign the following declaration:

I Hereby declare that all the work was done by myself. I was allowed to use Maple (unless specifically told not to), calculators, the book, and all the material in the web-page of this class but **not** other resources on the internet.

I only spent (at most) 3 hours on doing the exam. The last 30 minutes were spent in checking and double-checking the answers.

I also understand that I may be subject to a random short chat to verify that I actually did it all by myself.

Signed: Rahul Paleja

Possibly useful facts from school Geometry (that you are welcome to use) : (i) The area of a circle radius r is πr^2. (ii) The circumference of a circle radius r is $2\pi r$ (iii) The parametric equation of an ellipse with axes a b and parallel to the x and y axes respectively is $x = a\cos\theta$, $y = b\cos\theta$, $0 < \theta < 2\pi$. (iv) The area of an ellipse with axes a and b is πab (v) The volume and surface area of a sphere radius R are $\frac{4}{3}\pi R^3$ and $4\pi R^2$ respectively (vi) The volume of a cone is the area of the base times the height over 3. (vii) The volume of a pyramid is the area of the base times the height over 3. (viii) The area of a triangle is base times height over 2.

Formula that you may (or may not) need

If the surface S is given in **explicit** notation $z = g(x, y)$, above the region of the xy-plane , D, then

$$\int\int_S \mathbf{F} \cdot d\mathbf{S} =$$

$$\int\int_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right) \, dA \quad .$$

1. (12 pts.) **Without using Maple (or any software)** Compute the **vector-field line integral**

$$\int_C \overbrace{(\cos(e^{\sin x}) + 5y)}^{P}\,dx + \overbrace{(\sin(e^{\cos y}) + 11x)}^{Q}\,dy \quad ,$$

over the path consisiting of the five line segments (in that order)

$$(1,0) \to (-1,0) \to (-1,1) \to (0,2) \to (1,1) \to (1,0) \quad .$$

Explain!

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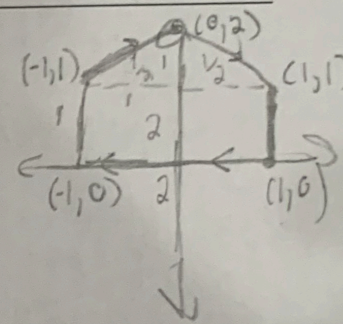
ans. -18

In the Form: $\int_C P\,dx + Q\,dy$



$= -\iint Q_x - P_y \, dA$

clockwise

$= -\iint_R 11 - 5 \, dA = -6 \iint_R dA$

$= -6 \cdot \text{Area} = -6(3) = \boxed{-18}$

2. (12 points) Change the order of integration

$$\int_{\frac{1}{4}}^{1}\int_{0}^{\sqrt{x}} f(x,y)\,\cancel{dx\,dy}\;dy\,dx\ .$$

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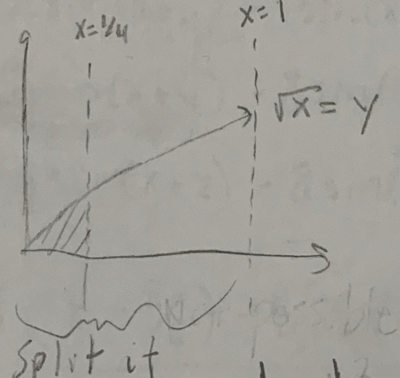
ans. $\int_0^1 \int_{y^2}^1 F(x,y)\,dx\,dy - \int_0^{1/2}\int_{y^2}^{1/4} F(x,y)\,dx\,dy$



You depend on the curve so you must split

$$\int_0^1\int_{y^2}^1 F(x,y)\,dx\,dy - \int_0^{1/2}\int_{y^2}^{1/4} F(x,y)\,dx\,dy$$

3. (12 points) Find the equation of the tangent plane at the point $(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6})$ to the surface given implicitly by

$$2\cos(x+y) + 4\cos(x+z) + 8\cos(y+z) = 7 \quad .$$

Express you answer in **explicit** form, i.e in the format $z = ax + by + c$.

ans. $z =$ $-\frac{x}{2} - \frac{5}{6}y + \frac{14\pi}{36}$

$$F_x(\tfrac{\pi}{6}, \tfrac{\pi}{6}, \tfrac{\pi}{6})(x - x_0) + F_y(\tfrac{\pi}{6}, \tfrac{\pi}{6}, \tfrac{\pi}{6})(y - y_0) + F_z(\tfrac{\pi}{6}, \tfrac{\pi}{6}, \tfrac{\pi}{6})(z - z_0) = 0$$

$$F_x = -2\sin(x+y) - 4\sin(x+z) \text{ at } (\tfrac{\pi}{6}, \tfrac{\pi}{6}, \tfrac{\pi}{6}) = -\sqrt{3} - 2\sqrt{3} = -3\sqrt{3}$$

$$F_y = -2\sin(x+y) - 8\sin(y+z) \text{ at } (\tfrac{\pi}{6}, \tfrac{\pi}{6}, \tfrac{\pi}{6}) = -\sqrt{3} - 4\sqrt{3} = -5\sqrt{3}$$

$$F_z = -4\sin(x+z) - 8\sin(y+z) \text{ at } (\tfrac{\pi}{6}, \tfrac{\pi}{6}, \tfrac{\pi}{6}) = -2\sqrt{3} - 4\sqrt{3} = -6\sqrt{3}$$

$$\frac{-3\cancel{\sqrt{3}}(x - \tfrac{\pi}{6}) - 5\sqrt{3}(y - \tfrac{\pi}{6}) - 6\sqrt{3}(z - \tfrac{\pi}{6})}{\cancel{\sqrt{3}}} = \frac{0}{\sqrt{3}}$$

$$-3x + \tfrac{\pi}{2} - 5y + \tfrac{5\pi}{6} - 6z + \pi = 0$$

$$\frac{6z}{6} = \frac{-3x - 5y + \frac{14\pi}{6}}{6}$$

$$\boxed{z = \frac{-x}{2} - \frac{5}{6}y + \frac{14\pi}{36}}$$

4. (16 points) Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three vectors such that

$$\mathbf{a} \times \mathbf{b} = \mathbf{i} + \mathbf{j} - \mathbf{k} \quad , \quad \mathbf{b} \times \mathbf{c} = \mathbf{i} - \mathbf{j} + \mathbf{k} \quad , \quad \mathbf{a} \times \mathbf{c} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} \quad .$$

What is

$$(\mathbf{a} + \mathbf{b} + \mathbf{c}) \times (2\mathbf{a} - \mathbf{b} + 3\mathbf{c}) \qquad ?$$

---

ans.

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5. (12 points) Find the three angles of the triangle ABC where

$$A = (0,0,0) \quad , \quad B = (1,0,1) \quad , \quad C = (1,1,0) \quad .$$

---

ans. The angle at A is: $\pi/3$ radians ;

The angle at B is: $\pi/3$ radians ;

The angle at C is: $\pi/3$ radians ;

$\cos\theta = \dfrac{a \cdot b}{|a||b|}$

$\vec{AB} = \langle 1,0,1 \rangle = a \quad |a| = \sqrt{2}$

$\vec{AC} = \langle 1,1,0 \rangle = b \quad |b| = \sqrt{2}$

$\cos\theta = \dfrac{1+0+0}{\sqrt{2}\cdot\sqrt{2}} = \left(\dfrac{1}{2} = \cos\theta\right) \cos^{-1}$

$\theta = \pi/3$

$\vec{BA} = \langle -1,0,-1 \rangle = a \quad |a| = \sqrt{2}$

$\vec{BC} = \langle 0,1,-1 \rangle = b \quad |b| = \sqrt{2}$

$\cos\theta = \dfrac{0+0+1}{\sqrt{2}\cdot\sqrt{2}} = \left(\dfrac{1}{2} = \cos\theta\right) \cos^{-1} \qquad \theta = \dfrac{\pi}{3}$

$\vec{CA} = \langle 1,1,0 \rangle = a \quad |a| = \sqrt{2}$

$\vec{CB} = \langle 0,-1,1 \rangle = b \quad |b| = \sqrt{2}$

$\cos\theta = \dfrac{0+1+0}{\sqrt{2}\cdot\sqrt{2}} = \left(\dfrac{1}{2} = \cos\theta\right) \cos^{-1} \qquad \theta = \pi/3$

6. (12 points) Find the directional derivative of

$$f(x, y, z) = x^3 + y^3 + z^3 + xyz \quad ,$$

at the point $(1, 1, 1)$ in a direction pointing to the point $(-1, -1, -1)$.

ans. $-4\sqrt{3}$

$\nabla f = \langle 3x^2 + yz, 3y^2 + xz, 3z^2 + xy \rangle$

$\nabla f(-1,-1,-1) = \langle 4, 4, 4 \rangle$

unit vector of direction

$|\langle -1,-1,-1 \rangle| = \sqrt{(-1)^2 + (-1)^2 + (-1)^2} = \sqrt{3}$

Unit vector of direction:

$u = \langle \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \rangle$

$u \cdot \nabla f = \langle \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \rangle \cdot \langle 4, 4, 4 \rangle$

$= \frac{-4}{\sqrt{3}} - \frac{4}{\sqrt{3}} - \frac{4}{\sqrt{3}} = \frac{-12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-12\sqrt{3}}{3}$

$= \boxed{-4\sqrt{3}}$

7. (12 points) Using the Chain Rule (no credit for other methods), find

$$\frac{\partial g}{\partial u}$$

at $(u, v) = (0, 1)$, where

$$g(x, y) = 3x^2 - 3y^2 \quad ,$$

and

$$x = e^u \cos v \quad , \quad y = e^u \sin v \quad .$$

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ans. $6(\cos^2(1) - \sin^2(1))$

$$\frac{\partial g}{\partial u} = \frac{\partial g}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial g}{\partial y} \cdot \frac{\partial y}{\partial u}$$

$$\frac{\partial g}{\partial x} = 6x \quad \frac{\partial x}{\partial u} = e^u \cos v \quad \frac{\partial g}{\partial y} = -6y \quad \frac{\partial y}{\partial u} = e^u \sin v$$

$$\frac{\partial g}{\partial u} = 6x \cdot e^u \cos v + (-6y)(e^u \sin v)$$

Plug in x and y

$$\frac{\partial g}{\partial u} = 6(e^u \cos v) \cdot e^u \cos v + (-6e^u \sin v)(e^u \sin v)$$

$$\frac{\partial g}{\partial u} = 6e^{2u} \cos^2 v - 6e^{2u} \sin^2 v$$

$$\frac{\partial g}{\partial u} \text{ at } (0,1) = 6\cos^2(1) - 6\sin^2(1)$$

$$= \boxed{6(\cos^2(1) - \sin^2(1))}$$

8. (12 points) Without using Maple (or any other software), compute the vector-field surface integral $\int_S \mathbf{F}.d\mathbf{S}$ if

$$\mathbf{F} = \langle 3x + cos(y^3 + yz) \,,\, -2y + e^{x+z^2} \,,\, 5z + \sin(xy^3 + e^x) \rangle \quad ,$$

and S is the closed surface in 3D space bounding the region

$$\{(x, y, z) : x^2 + y^2 + z^2 < 4 \quad and \quad x > 0 \quad and \quad y < 0 \quad and \quad z > 0\} \quad .$$

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ans.

$$\int_{3\pi/2}^{2\pi} \int_0^2 -(3x + \cos(y^3 + yz))\,\partial x - (-2y + e^{x+z^2})\,\partial y + (5z + \sin(xy^3 + e^x))\, r\,dr\,d\theta$$

9. (12 points) Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if

$$\mathbf{F} = \langle 3z\,,\, 2x\,,\, y+z\rangle \quad ,$$

and S is the oriented surface

$$z = 2x+3y \quad , \quad 0<x<1, \quad 0<y<1 \quad ,$$

with **upward pointing** normal.

ans. -15

$$\iint_S F\cdot ds = \iint_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right) dA$$

Plug z into F: $< 6x+9y, 2x, 2x+4y >$

$$\frac{\partial g}{\partial x} = 2 \qquad \frac{\partial g}{\partial y} = 3$$

$$\iint_D \left(-(6x+9y)(2) - (2x)(3) + 2x+4y\right) dA$$

$0<x<1\,,\ 0<y<1$

$$\int_0^1\int_0^1 (-12x-18y-6x+2x+4y)\,dx\,dy$$

$$\int_0^1\int_0^1 (-16x-14y)\,dx\,dy$$

Inner:

$$\int_0^1 (-16x-14y)\,dx = -8x^2-14xy\Big]_0^1 = -8-14y$$

Outer: $\displaystyle\int_0^1 -8-14y\,dy = -8y-7y^2\Big]_0^1 = -8-7 = \boxed{-15}$

10. (12 points) Without using Maple or software, find the critical point(s) of

$$f(x,y) = 4x - y^2 - \ln(2x+y) \quad ,$$

and decide for each whether it is a local maximum, local minimum, or saddle point. Explain.

ans. $(3/4, -1)$ is a saddle point

$F_x = 4 - \frac{2}{2x+y}$ $\quad$ $F_y = -2y - \frac{1}{2x+y}$

$F_{xx} = \frac{2\cdot 2}{(2x+y)^2} = \frac{4}{(2x+y)^2}$ $\quad$ $F_{yy} = -2 + \frac{1}{(2x+y)^2}$

$F_{xy} = \frac{-8}{(2x+y)^3}$

$F_{xx}(3/4, -1) = \frac{4}{(\frac{1}{2})^2} = \frac{4}{1/4} = 16$

$F_{yy}(3/4, -1) = -2 + \frac{1}{(\frac{1}{2})^2} = -2 + 4 = 2$

$F_{xy}(3/4, -1) = \frac{-8}{(\frac{1}{2})^3} = \frac{-8}{\frac{1}{8}} = -64$

$F_x = 0$ and $F_y = 0$

$4 - \frac{2}{2x+y} = 0$ $\quad$ $-2y - \frac{1}{2x+y} = 0$

$4 = \frac{2}{2x+y}$ $\quad$ $-2(\frac{1}{2} - 2x) - \frac{1}{2x + (\frac{1}{2} - 2x)} = 0$

$8x + 4y = 2$ $\quad$ $-1 + 4x - \frac{1}{\cancel{2x} + \frac{1}{2} - \cancel{2x}} = 0$

$\frac{4y}{4} = \frac{2 - 8x}{4}$ $\quad$ $-1 + 4x - 2 = 0$

$y = \frac{1}{2} - 2x$ $\quad$ $-3 + 4x = 0$

$y = \frac{1}{2} - \frac{\cancel{8}^{2}}{1}\left(\frac{3}{\cancel{4}}\right) = \frac{1}{2} - \frac{3}{2} = \frac{-2}{2} = \boxed{-1}$ $\quad$ $\frac{4x}{4} = \frac{3}{4}$

$x = 3/4$

Critical Point: $(3/4, -1)$

$D = 16(2) - |-64|^2 \rightarrow$ Negative

$(3/4, -1)$ is a saddle point

11. (12 points) Without using Maple or software, using a **Linearization** around the point $(1, 1, 2)$, approximate $f(1.001, 0.999, 2.001)$ if

$$f(x, y, z) = \sqrt{2x^2 + 3y^2 + z^2} \quad .$$

ans.

$$F(1,1,2) = \sqrt{2+3+4} = 3$$

$$F_x = \frac{\cancel{2}\,4x}{\cancel{2}\sqrt{2x^2+3y^2+z^2}} \qquad F_y = \frac{6y}{2\sqrt{2x^2+3y^2+z^2}} \qquad F_z = \frac{\cancel{2}z}{\cancel{2}\sqrt{2x^2+3y^2+z^2}}$$

$$F_x(1,1,2) = \frac{2}{\sqrt{2+3+4}} \qquad F_y(1,1,2) = \frac{6}{2\sqrt{2+3+4}} \qquad F_z(1,1,2) = \frac{2}{\sqrt{2+3+4}}$$

$$F_x(1,1,2) = \frac{2}{3} \qquad F_y(1,1,2) = \frac{6}{6} = 1 \qquad F_z(1,1,2) = \frac{2}{3}$$

$$\text{Linearization} = 3 + \frac{2}{3}(x-1) + 1(y-1) + \frac{2}{3}(z-2)$$

$$F(1.001 = \cancel{3} + \frac{2}{3}x - \cancel{\frac{2}{3}} + y - \cancel{1} + \frac{2}{3}z - \frac{4}{3}$$

$$\text{Linearization} = \frac{2}{3}x + y + \frac{2}{3}z$$

12. (12 points) Without using Maple (or any other software) and by using polar coordinates (no credit for doing it directly) find

$$\int_0^{\frac{\sqrt{2}}{2}} \int_0^{x} x\,dy\,dx + \int_{\frac{\sqrt{2}}{2}}^{1} \int_0^{\sqrt{1-x^2}} x\,dy\,dx \quad .$$

Explain!

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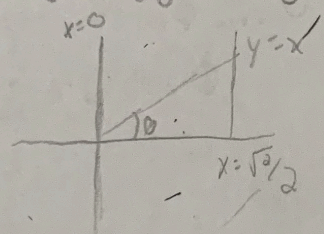
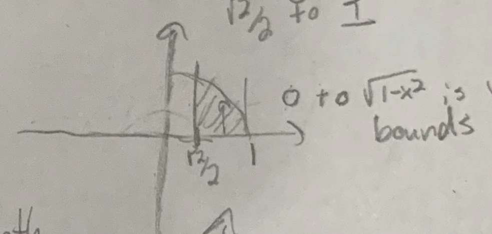
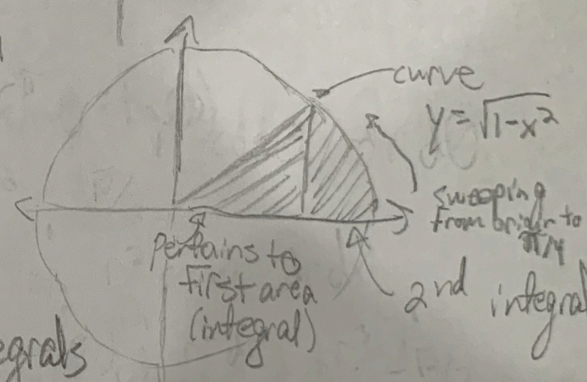
ans. $\frac{\sqrt{2}}{6}$

$\int_0^{\sqrt{2}/2} \int_0^{x} x\,dydx$ $\qquad$ $\int_{\sqrt{2}/2}^{1} \int_0^{\sqrt{1-x^2}} x\,dydx$





We can combine both drawings



Thus, combining both integrals we can form a single integral:

→ Not doing entire circle

$$\int_0^{\pi/4} \int_0^1 r\cos\theta \; r\,dr\,d\theta = \frac{1}{3\sqrt{2}}$$

x → same integrands in both

inner integral: $\cos\theta \int_0^1 r^2\,dr = \frac{r^3}{3}\Big]_0^1 = \frac{1}{3}$

Outer Integral: $\frac{1}{3}\int_0^{\pi/4} \cos\theta\,d\theta = \frac{1}{3}(\sin\theta\Big]_0^{\pi/4} = \frac{1}{3}\cdot\frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2}}{6}}$

13. (12 points) Convert the triple iterated integral

$$\int_0^4 \int_0^{\sqrt{4-z^2}} \int_{-\sqrt{4-z^2-y^2}}^0 x^2\, y\, z\, dx\, dy\, dz$$

to **spherical coordinates. Do not evaluate.**

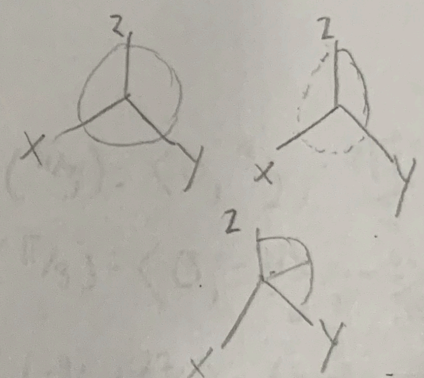
ans. $\int_0^{\pi/2} \int_{\pi/2}^{\pi} \int_0^{\sqrt{2}} \rho^6 \sin^4\phi \cos\phi \cos^2\theta \sin\theta \, d\rho\, d\theta\, d\phi$

$x = 2\sin\phi\cos\theta$

$y = 2\sin\phi\sin\theta$

$z = 2\cos\phi$

$\rho = 2$



$$\int_0^{\pi/2} \int_{\pi/2}^{\pi} \int_0^{\sqrt{2}} (\rho^2 \sin^2\phi \cos^2\theta)(\rho \sin\phi \sin\theta)(\rho\cos\phi)\, \rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$$

$$\int_0^{\pi/2} \int_{\pi/2}^{\pi} \int_0^{\sqrt{2}} \rho^6 \sin^4\phi \cos\phi \cos^2\theta \sin\theta \, d\rho\, d\theta\, d\phi$$

14. (12 points) Find the curvature of the curve

$$\mathbf{r}(t) = \langle 5, 3\sin t, 3\cos t\rangle$$

at the point where $t = \frac{\pi}{3}$.

ans. $1/3$

$$\kappa(t) = \frac{|r'(t) \times r''(t)|}{|r'(t)|^3}$$

$r'(t) = \langle 0, 3\cos t, -3\sin t\rangle$ $\quad r'(\pi/3) = \langle 0, 3/2, -\frac{3\sqrt{3}}{2}\rangle$

$r''(t) = \langle 0, -3\sin t, -3\cos t\rangle$ $\quad r''(\pi/3) = \langle 0, -\frac{3\sqrt{3}}{2}, -3/2\rangle$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 0 & 3/2 & -3\sqrt{3}/2 \\ 0 & -3\sqrt{3}/2 & -3/2 \end{vmatrix}$$

$i(-9/4 - 27/4) - j(0-0) + k(0-0)$

$i(-36/4) - j(0) + k(0)$

$= -9i - 0j + 0k$

$|r'(t) \times r''(t)| = \sqrt{(-9)^2 + (0)^2 + (0)^2} = 9$

$|r'(t)| = \sqrt{(\frac{3}{2})^2 + (\frac{-3\sqrt{3}}{2})^2} = \sqrt{\frac{9}{4} + (\frac{27}{4})} = \sqrt{9} = 3$

$|r'(t)|^3 = 27$

$$K(\tfrac{\pi}{3}) = \frac{9}{27} = \frac{1}{3}$$

15. (12 points) Set-up an iterated double integral, in type I format, but do not compute, for the surface area of the surface given parameterically by

$$\mathbf{r}(u,v) = \langle u^2, uv, v^2 \rangle \quad , \quad 0 < u < v < 1 \quad .$$

ans. $\int_0^1 \int_u^1 F(r(u,v)) \cdot \sqrt{4v^4 + 16v^2u^2 + 4u^4} \, dv \, du$

$$\int_0^1 \int_u^1 \quad dv \, du$$

$r_u = 2u, v, 0$

$r_v = 0, u, 2v$

$$|r_u \times r_v| = \begin{vmatrix} i & j & k \\ 2u & v & 0 \\ 0 & u & 2v \end{vmatrix}$$

$$= i(2v^2) - j(4uv) + k(2u^2)$$

Magnitude: $\sqrt{4v^4 + 16v^2u^2 + 4u^4}$

arbitrary function as r(u,v) is parametric representation

$$\int_0^1 \int_u^1 F(r(u,v)) \cdot \sqrt{4v^4 + 16v^2u^2 + 4u^4} \, dv \, du$$

16. (12 points) Let

$$f(x,y,z) = xy^2z^3 \quad ,$$

and let

$$g(x,y,z) = x + y^2 + z^3 \quad .$$

compute the dot-product

$$grad(f) \,.\, grad(g) \quad .$$

at the point $(1,1,1)$.

---

ans. 14

$Df = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$

Df at $(1,1,1)$

$= \langle 1, 2, 3 \rangle$

$Dg = \langle 1, 2y, 3z^2 \rangle$

Dg at $(1,1,1)$

$= \langle 1, 2, 3 \rangle$

$\langle 1,2,3 \rangle \cdot \langle 1,2,3 \rangle$

$= 1 + 4 + 9 = 14$

17. (8 points) Decide whether the following limit exists. If it does ,find it. If it does not, explain why it does not exist.

$$\lim_{(x,y,z,w)\to(0,0,0,0)} \frac{(x+y)^2-(z+w)^2}{x+y-z-w} = 0$$

---

ans. 0

Keep $y=0$, $z=0$, $w=0$ $\rightarrow \lim_{x\to 0} \frac{x^{\cancel{2}}}{\cancel{x}} = 0$

$x=0$, $z=0$, $w=0$ $\rightarrow \lim_{y\to 0} \frac{y^{\cancel{2}}}{\cancel{y}} = 0$

$x=0$, $y=0$, $w=0$ $\rightarrow \lim_{z\to 0} \frac{\cancel{-}z^{\cancel{2}\,1}}{\cancel{-}\cancel{z}} = 0$

$x=0$, $y=0$, $z=0$ $\rightarrow \lim_{w\to 0} \frac{\cancel{-}w^{\cancel{2}}}{\cancel{-}\cancel{w}} = 0$

By holding certain variables constant and analyzing how we approach (0,0,0,0) we determine that this limit = 0