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SSC: (circle) None / I / II / I and II

MATH 251 (22,23,24) [Fall 2020], Dr. Z. , Final Exam , Tue., Dec. 15, 2020

Email the completed test, renamed as `finalFirstLast.pdf` to DrZcalc3@gmail.com no later than 3:30pm, (or, in case of conflict, three and half hours after the start).

WRITE YOUR FINAL ANSWERS BELOW

1.

2. $\int_0^1 \int_0^{y^2} f(x,y)\,dx\,dy$

3. $z = -3x - 5y + \frac{7\pi}{3}$

4. 0

5. Angle at A is $\frac{\pi}{3}$ radians, at B is $-\frac{\pi}{3}$ radians, at C is $\frac{\pi}{3}$ radians

6. $D_u f(1,1,1) = \frac{-12}{\sqrt{3}}$

7. $6\cos 2(1) - 6\sin 2(1)$

8. 8π

9. -15

10. since $D > 0$ and $f_{xx} < 0$, $(\frac{1}{4}, 0)$ and $(\frac{9}{4}, -4)$ are localmaximum points.

11. $f(1.001, 0.999, 2.001) \approx 0.334$

12. $\frac{1}{3\sqrt{2}}$ because the two double integrals turn to $\frac{1}{6\sqrt{2}}$ and $2(\frac{1}{6\sqrt{2}})$ is $\frac{1}{3\sqrt{2}}$.

13. $\int_0^2 \int_0^{\pi} \int_{\pi/2}^{\pi} \rho^6 \sin^4\phi \cos^2\theta \sin\theta \cos\phi \, d\phi\, d\theta\, d\rho$

14. $K = \frac{1}{3}$

15. $\int_0^1 \int_v^1 (\sqrt{4v^4 + 16v^2u^2 + 4u^4})\,dv\,du$

16. $\text{grad} f \cdot \text{grad} g = 14$

17.

Sign the following declaration:

I Rachel Balji Hereby declare that all the work was done by myself. I was allowed to use Maple (unless specifically told not to), calculators, the book, and all the material in the web-page of this class but **not** other resources on the internet.

I only spent (at most) 3 hours on doing the exam. The last 30 minutes were spent in checking and double-checking the answers.

I also understand that I may be subject to a random short chat to verify that I actually did it all by myself.

Signed: Rachel Balji

Possibly useful facts from school Geometry (that you are welcome to use) : (i) The area of a circle radius r is πr^2. (ii) The circumference of a circle radius r is $2\pi r$ (iii) The parametric equation of an ellipse with axes a b and parallel to the x and y axes respectively is $x = a\cos\theta$, $y = b\cos\theta$, $0 < \theta < 2\pi$. (iv) The area of an ellipse with axes a and b is πab (v) The volume and surface area of a sphere radius R are $\frac{4}{3}\pi R^3$ and $4\pi R^2$ respectively (vi) The volume of a cone is the area of the base times the height over 3. (vii) The volume of a pyramid is the area of the base times the height over 3. (viii) The area of a triangle is base times height over 2.

Formula that you may (or may not) need

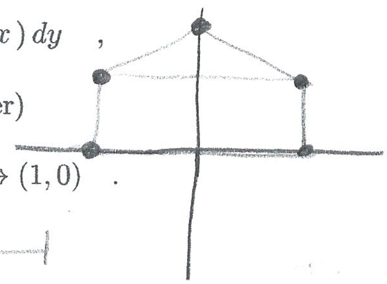
If the surface S is given in **explicit** notation $z = g(x, y)$, above the region of the xy-plane , D, then

$$\int\int_S \mathbf{F} \cdot d\mathbf{S} =$$

$$\int\int_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right)\, dA \quad .$$

1. (12 pts.) **Without using Maple (or any software)** Compute the **vector-field line integral**

$$\int_C \overset{P}{\left(\cos\left(e^{\sin x}\right) + 5y\right)}\, dx \;+\; \overset{Q}{\left(\sin\left(e^{\cos y}\right) + 11x\right)}\, dy \quad ,$$



over the path consisiting of the five line segments (in that order)

$$(1,0) \to (-1,0) \to (-1,1) \to (0,2) \to (1,1) \to (1,0) \quad .$$

Explain!

---

ans.

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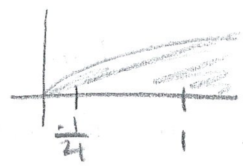
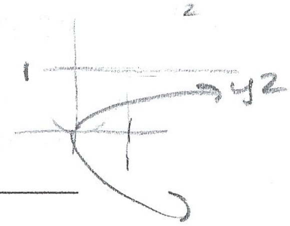
2. (12 points) Change the order of integration

$$\int_{\frac{1}{4}}^{1}\int_{0}^{\sqrt{x}} f(x,y)\,dx\,dy$$





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ans. $\int_0^1 \int_0^{y^2} f(x,y)\,dx\,dy$

$\int_{\frac{1}{4}}^{1} \int_0^{\sqrt{x}} f(x,y)\,dy\,dx$

$y = \sqrt{x}$

$x = y^2$

$\int_0^1 \int_0^{y^2} f(x,y)\,dx\,dy$

3. (12 points) Find the equation of the tangent plane at the point $(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6})$ to the surface given implicitly by

$$2\cos(x+y) + 4\cos(x+z) + 8\cos(y+z) = 7 \quad .$$

Express you answer in **explicit** form, i.e in the format $z = ax + by + c$.

---

ans. $z = -3x - 5y + \frac{7\pi}{3}$

$2\cos(x+y) + 4\cos(x+z) + 8\cos(y+z) = 7$

$F(x,y,z) = 2\cos(x+y) + 4\cos(x+z) + 8\cos(y+z) - 7$

$@(\pi/6, \pi/6, \pi/6) = 2(\frac{1}{2}) + 4(\frac{1}{2}) + 8(\frac{1}{2}) - 7$

$\hookrightarrow 30°$ $\quad = 1 + 2 + 4 - 7 = 0$

$f_x = -2\sin(x+y) - 4\sin(x+z) \quad @P = -2(\frac{\sqrt{3}}{2}) - 4(\frac{\sqrt{3}}{2})$
$= -\sqrt{3} - 2\sqrt{3} = -3\sqrt{3}$

$f_y = -2\sin(x+y) - 8\sin(y+z) \quad @P = -2(\sqrt{3}/2) - 8(\sqrt{3}/2)$
$= -\sqrt{3} - 4\sqrt{3} = -5\sqrt{3}$

$f_z = -4\sin(x+z) - 8\sin(y+z) \quad @P = -4(\sqrt{3}/2) - 8(\sqrt{3}/2)$
$= -2\sqrt{3} - 4\sqrt{3} = -6\sqrt{3}$

Eq. @ the tan plane:

$-3\sqrt{3}(x - \frac{\pi}{6}) - 5\sqrt{3}(y - \frac{\pi}{6}) - 6\sqrt{3}(z - \frac{\pi}{6}) = 0$

$-3\sqrt{3}x + \frac{\sqrt{3}\pi}{2} - 5\sqrt{3}y + \frac{5\sqrt{3}\pi}{6} - 6\sqrt{3}z + \sqrt{3}\pi = 0$

$-6\sqrt{3}\, z = -\sqrt{3}\pi + 3\sqrt{3}x - \frac{\sqrt{3}\pi}{2} + 5\sqrt{3}y - \frac{5\sqrt{3}\pi}{6}$

$z = \dfrac{\cancel{\sqrt{3}}\pi - 3\cancel{\sqrt{3}}x + \frac{\cancel{\sqrt{3}}\pi}{2} - 5\cancel{\sqrt{3}}y + \frac{5\cancel{\sqrt{3}}\pi}{6}}{6\cancel{\sqrt{3}}}$

$\frac{\pi}{2} = \frac{3\pi}{6}$

$\frac{8\pi}{6} = \frac{14\pi}{6}$

$z = \pi - 3x + \frac{\pi}{2} - 5y + \frac{5\pi}{6}$

$\boxed{z = \frac{14\pi}{6} - 3x - 5y} \rightarrow \boxed{z = \frac{7\pi}{3} - 3x - 5y}$

4. (16 points) Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three vectors such that

$$\mathbf{a} \times \mathbf{b} = \mathbf{i} + \mathbf{j} - \mathbf{k} \quad , \quad \mathbf{b} \times \mathbf{c} = \mathbf{i} - \mathbf{j} + \mathbf{k} \quad , \quad \mathbf{a} \times \mathbf{c} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} \quad .$$

What is

$$(\mathbf{a} + \mathbf{b} + \mathbf{c}) \times (2\mathbf{a} - \mathbf{b} + 3\mathbf{c}) \qquad ?$$

---

ans. 0

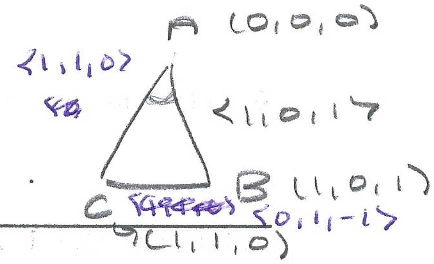
$a \times b = i + j - k = \langle 1, 1, -1 \rangle$ $(a+b+c) \times (2a - b + 3c)$

$b \times c = i - j + k$ $\langle 1, -1, 1 \rangle$

$a \times c = 2i + j + 2k$ $\langle 2, 1, 2 \rangle$

5. (12 points) Find the three angles of the triangle ABC where

$$A = (0,0,0) \quad , \quad B = (1,0,1) \quad , \quad C = (1,1,0) \quad .$$



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ans. The angle at A is: $\frac{\pi}{3}$ radians ;   60

The angle at B is: $\frac{-\pi}{3}$ radians ;   60   120

The angle at C is: $\frac{\pi}{3}$ radians ;

$\rightarrow \overrightarrow{AB} = (1,0,1) - (0,0,0) = \langle 1,0,1 \rangle$

$\rightarrow \overrightarrow{AC} = (1,1,0) - (0,0,0) = \langle 1,1,0 \rangle$

$\overrightarrow{BC} = (1,1,0) - (1,0,1) = \langle 0,1,-1 \rangle$

$\measuredangle A \Rightarrow \cos\theta = \frac{AB \cdot AC}{(|AB||AC|)} = \frac{(1+0+0)}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2} \longrightarrow \cos\theta = \frac{1}{2} \quad \theta = 60° = \frac{\pi}{3}$

$\measuredangle B \Rightarrow \cos\theta = \frac{AB \cdot BC}{(|AB||BC|)} = \frac{-1}{\sqrt{4}} = \frac{-1}{2} \longrightarrow \cos\theta = -\frac{1}{2} \rightarrow \theta = -\frac{\pi}{3}$

$\measuredangle C \Rightarrow \cos\theta = \frac{BC \cdot AC}{(|BC||AC|)} = \frac{0+1+0}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{\sqrt{4}} = \frac{1}{2} \quad \cos\theta = \frac{1}{2} \longrightarrow \theta = \frac{\pi}{3}$

6. (12 points) Find the directional derivative of

$$f(x, y, z) = x^3 + y^3 + z^3 + xyz \quad ,$$

at the point $(1, 1, 1)$ in a direction pointing to the point $(-1, -1, -1)$.

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ans. $D_u f(1,1,1) = \dfrac{-12}{\sqrt{3}}$

$f(x,y,z) = x^3 + y^3 + z^3 + xyz$

Direction of vector $v = (-1-1 \ , -1-1 \ , -1-1) = \langle -2, -2, -2 \rangle$

$f_x = 3x^2 + yz$ @ $P = 4$

$f_y = 3y^2 + xz$ @ $P = 4$ $\Big\} \rightarrow$ $\nabla f = \langle 4, 4, 4 \rangle$

$f_z = 3z^2 + xy$ @ $P = 4$

$\dfrac{1}{\|v\|} = \sqrt{4+4+4}$
$= \sqrt{12} = 2\sqrt{3}$

$$D_u f(1,1,1) = \frac{1}{\|\langle -2,-2,-2 \rangle\|} \nabla f_P \cdot v$$

$$= \frac{1}{2\sqrt{3}} \langle 4,4,4 \rangle \cdot \langle -2,-2,-2 \rangle$$

$$= \langle \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}} \rangle \cdot \langle -2,-2,-2 \rangle$$

$$= \frac{-4}{\sqrt{3}} - \frac{4}{\sqrt{3}} - \frac{4}{\sqrt{3}} = \frac{-12}{\sqrt{3}}$$

7. (12 points) Using the Chain Rule (no credit for other methods), find

$$\frac{\partial g}{\partial u}$$

at $(u, v) = (0, 1)$, where

$$g(x, y) = 3x^2 - 3y^2 \quad ,$$

and

$$x = e^u \cos v \quad , \quad y = e^u \sin v \quad .$$

---

ans. $6\cos^2(1) - 6\sin^2(1)$

$g(x,y) = 3x^2 - 3y^2$

$x = e^u \cos v$
$y = e^u \sin v$

$\frac{\partial g}{\partial u} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial u}$

$x|_{(0,1)} = \cos(1)$

$\frac{\partial g}{\partial x} = 6x \quad \frac{\partial x}{\partial u} = e^u \cos v$

$y|_{(0,1)} = \sin(1)$

$\frac{\partial g}{\partial y} = -6y \quad \frac{\partial y}{\partial u} = e^u \sin v$

$\frac{\partial g}{\partial u} = 6x\,(e^u \cos v) + -6y\,(e^u \sin v)$

@ $P_{(0,1)} = \frac{\partial g}{\partial u} = 6x\,(\cos(1)) - 6y\,(\sin(1))$

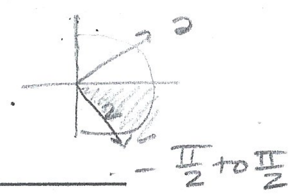
$= 6\cos^2(1) - 6\sin^2(1)$

8. (12 points) Without using Maple (or any other software), compute the vector-field surface integral $\int_S \mathbf{F}.d\mathbf{S}$ if

$$\mathbf{F} = \langle\, 3x + cos(y^3 + yz)\,,\, -2y + e^{x+z^2}\,,\, 5z + \sin(xy^3 + e^x)\,\rangle \quad ,$$

and S is the closed surface in 3D space bounding the region

$$\{(x, y, z) : x^2 + y^2 + z^2 < 4 \quad and \quad x > 0 \quad and \quad y < 0 \quad and \quad z > 0\} \; .$$



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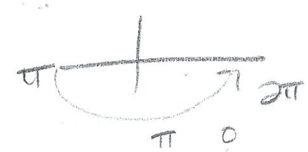
ans. 8π

$F = \langle 3x + \cos(y^3 + yz), -2y + e^{x+z^2}, 5z + \sin(xy^3 + e^x)\rangle$



$\text{div} F = 3 - 2 + 5 = 6$

$6 \int_0^2 \int_0^{\frac{\pi}{2}} \int_0^{\pi/2} p^2 \sin\phi \, d\phi \, d\theta \, dp$

$\int_0^{\pi/2} \sin\phi \, d\phi = -\cos\phi \Big|_0^{\pi/2} = -0 + 1 = 1$

$\int_0^{\pi/2} d\theta = \frac{\pi}{2}$

$6 \int_0^2 \pi/2 \; p^2 \, dp = 3\pi \int_0^2 p^2 \, dp$

$= 3\pi \left[\frac{p^3}{3}\right]\Big|_0^2$

$= \frac{\cancel{3}\pi\,(8)}{\cancel{3}} = 8\pi$

9. (12 points) Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if

$$\mathbf{F} = \langle \overset{P}{3z}\,,\, \overset{Q}{2x}\,,\, \overset{R}{y+z} \rangle \quad ,$$

and S is the oriented surface

$$z = 2x + 3y \quad , \quad 0 < x < 1, \quad 0 < y < 1 \quad ,$$

with **upward pointing** normal.

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ans. -15

$F = \langle 3z, 2x, y+z \rangle$

$z = 2x + 3y$
$0 < x < 1$
$0 < y < 1$

$$\iint_S F\cdot ds = \iint_D (-3z(2) - 2x(3) + y + z)\,dA$$

$$= \iint_D (-5z - 6x + y)\,dA$$

$$= \int_0^1\int_0^1 (-10x - 15y - 6x + y)\,dx\,dy$$

$$= \int_0^1\int_0^1 (-16x - 14y)\,dx\,dy$$

$$= -2\int_0^1\int_0^1 (8x + 7y)\,dx\,dy$$

$$\int_0^1 (8x+7y)\,dx = 4x^2 + 7yx\Big|_0^1 = 4 + 7y$$

$$= -2\int_0^1 [4 + 7y]\,dy$$

$$= -2\left[4y + \frac{7y^2}{2}\right]\Big|_0^1$$

$$= -2\left(4 + \frac{7}{2}\right) = -8 - 7 = -15$$

10. (12 points) Without using Maple or software, find the critical point(s) of

$$f(x,y) = 4x - y^2 - \ln(2x+y) \quad ,$$

and decide for each whether it is a local maximum, local minimum, or saddle point. Explain.

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ans. Since $D > 0$ and $f_{xx} < 0$, $(\frac{1}{4}, 0)$ and $(\frac{9}{4}, -4)$ are local max.

$f(x,y) = 4x - y^2 - \ln(2x+y)$

$f_x = 4 - \frac{2}{2x+y} = 0 \longrightarrow -\frac{2}{2x+y} = -4 \rightarrow 2 = 8x + 4y$

$f_y = -2y - \frac{1}{2x+y} = 0 \longrightarrow -2y - (2x+y)^{-1} = 0$

$-\frac{1}{(2x+y)} = 2y$

$-1 = 2y(2x+y)$

$-1 = 2\cancel{/}xy + 2y^2$

$0 = 2y^2 + 4xy - 1$

$f_{xx} = -\frac{4}{(2x+y)^2}$

$f_{xy} = -\frac{2}{(2x+y)^2}$

$f_{yy} = -2 - \frac{1}{(2x+y)^2}$

$4y = 2 - 8x$

$y = \frac{1}{2} - 2x$

$0 = (\frac{1}{2} - 2x)^2 + 4(\frac{1}{2} - 2x) - 1$

$0 = 4x^2 - 2x + \frac{1}{4} + 2 - 8x - 4$

$0 = 4x^2 - 10x + 2\frac{1}{4}$ -2

$x = \frac{1}{4}, \frac{9}{4}$

Points $\rightarrow$ $(\frac{1}{4}, 0)$ P_1

$(\frac{9}{4}, -4)$ P_2

$\cancel{2}\frac{9}{\cancel{4}} = \frac{9}{2}$

$\frac{1}{2} - \frac{9}{2} = -\frac{8}{2} = -4$

f_{xx} at $P_1 = -16$

f_{xx} at $P_2 = -16$

f_{yy} at $P_1 = -6$

f_{xy} at $P_1 = -8$

$D = f_{xx} f_{yy} - [f_{xy}]^2$

$= -16(-6) - [-8]^2$

$D = 96 - 64 = 32$

$D > 0$ and $f_{xx} < 0$

then $(\frac{1}{4}, 0)$ and $(\frac{9}{4}, -4)$ are local max

11. (12 points) Without using Maple or software, using a **Linearization** around the point $(1, 1, 2)$, approximate $f(1.001, 0.999, 2.001)$ if

$$f(x, y, z) = \sqrt{2x^2 + 3y^2 + z^2} \quad .$$

---

ans. $f(1.001, 0.999, 2.001) \approx 0.334$

$$f(x,y,z) = \sqrt{2x^2+3y^2+z^2} = (2x^2+3y^2+z^2)^{\frac{1}{2}} @P = \frac{1}{3}$$

$$f_x = \frac{1}{2} 4x(2x^2+3y^2+z^2)^{-\frac{1}{2}} = 2x(2x^2+3y^2+z^2)^{-\frac{1}{2}} = 2\left(\frac{1}{\sqrt{9}}\right) = \frac{2}{3}$$

$$f_y = \frac{1}{2} 6y(2x^2+3y^2+z^2)^{-\frac{1}{2}} = 3y(2x^2+3y^2+z^2)^{-\frac{1}{2}} = \frac{3}{3} = 1$$

$$f_z = \frac{1}{\cancel{2}} \cancel{2} z(2x^2+3y^2+z^2)^{-\frac{1}{2}} = \frac{2}{3}$$

$$L(x,y,z) = \frac{1}{3} + \frac{2}{3}(x-1) + (y-1) + \frac{2}{3}(z-2)$$

$$L(1.001, 0.999, 2.001) \approx \frac{1}{3} + \frac{2}{3}(0.001) + (0.999-1) + \frac{2}{3}(0.001)$$

$$\approx 0.334$$

12. (12 points) Without using Maple (or any other software) and by using polar coordinates (no credit for doing it directly) find

$$\int_0^{\frac{\sqrt{2}}{2}} \int_0^{x} x\,dy\,dx + \int_{\frac{\sqrt{2}}{2}}^{1} \int_0^{\sqrt{1-x^2}} x\,dy\,dx \quad .$$

Explain!

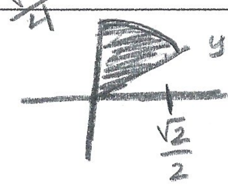
ans. $\frac{1}{3\sqrt{2}}$ because $\int_0^{\sqrt{2}/2}\int_0^{x} x\,dy\,dx$ turns to $\frac{1}{6\sqrt{2}}$ and so does $\int_{\sqrt{2}/2}^{1}\int_0^{\sqrt{1-x^2}} x\,dy\,dx$ when using polar coordinates

$$\int_0^{\sqrt{2}/2}\int_0^{x} x\,dy\,dx$$

$$= \int_0^{x} x\,dy = xy\Big|_0^{x} = x^2 - 0$$

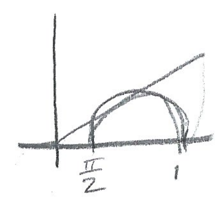
$$= x^2$$

$$\left(\frac{\sqrt{2}}{2}\right)^3 = \frac{2\sqrt{2}}{8} = \frac{\sqrt{2}}{4}$$



$$\int_0^{\sqrt{2}/2} x^2\,dx \Rightarrow \frac{x^3}{3}\Big|_0^{\frac{\sqrt{2}}{2}}$$

$$= \frac{1}{3\cdot 2^{2/3}}$$



$$\int_0^{\pi/4}\int_0^{1} r^2\cos\theta\,dr\,d\theta$$

$$= \int_0^1 r^2\,dr$$

$$= \frac{r^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$\frac{1}{3}\int_0^{\pi/4}\cos\theta\,d\theta$$

$$= \frac{1}{2}\frac{1}{3}\sin\theta\Big|_0^{\pi/4} = \frac{1}{3}\cdot\frac{1}{\sqrt{2}}\cdot\frac{1}{2}$$

$$= \frac{1}{6\sqrt{2}}$$

$$\int_0^{\sqrt{2}/2}\int_0^{x} x\,dy\,dx + \int_{\sqrt{2}/2}^{1}\int_0^{\sqrt{1-x^2}} x\,dy\,dx$$

$$\frac{1}{6\sqrt{2}} + \frac{1}{6\sqrt{2}} = \frac{2}{6\sqrt{2}} = \frac{1}{3\sqrt{2}}$$

top half so

13. (12 points) Convert the triple iterated integral

$$\int_0^4 \int_0^{\sqrt{4-z^2}} \int_{-\sqrt{4-z^2-y^2}}^{0} x^2\, y\, z\, dx\, dy\, dz$$

to **spherical coordinates. Do not evaluate.**

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ans. $\int_0^2 \int_0^{\pi} \int_{\frac{\pi}{2}}^{\pi} \rho^6 \sin^4\phi \cos^2\theta \sin\theta \cos\phi \, d\phi\, d\theta\, d\rho$

$$\int_0^4 \int_0^{\sqrt{4-z^2}} \int_{-\sqrt{4-z^2-y^2}}^{0} x^2 y z \, dx\, dy\, dz$$

$0 \le \rho \le 2$

$0 \le \theta \le \pi$

$\frac{\pi}{2} \le \phi \le \pi$

$$\int_0^2 \int_0^{\pi} \int_{\frac{\pi}{2}}^{\pi} (\rho\sin\phi\cos\theta)^2(\rho\sin\phi\sin\theta)(\rho\cos\phi)(\rho)^2\sin\phi \, d\phi\, d\theta\, d\rho$$

14. (12 points) Find the curvature of the curve

$$\mathbf{r}(t) = \langle 5, 3\sin t, 3\cos t\rangle$$

at the point where $t = \frac{\pi}{3}$.

ans. $\kappa = \frac{1}{3}$.

$$\kappa = \frac{|r' \times r''|}{|r'|^3}$$

$r(t) = \langle 5, 3\sin t, 3\cos t\rangle$

$r'(t) = \langle 0, 3\cos t, -3\sin t\rangle$

$r''(t) = \langle 0, -3\sin t, -3\cos t\rangle$

$r(\pi/3) = \langle 5, \frac{3\sqrt{3}}{2}, \frac{3}{2}\rangle$

$r'(\pi/3) = \langle 0, \frac{3}{2}, -\frac{3\sqrt{3}}{2}\rangle$

$r''(\pi/3) = \langle 0, -\frac{3\sqrt{3}}{2}, -3/2\rangle$

$$|r' \times r''| = \begin{vmatrix} 0 & \frac{3}{2} & -\frac{3\sqrt{3}}{2} \\ 0 & -\frac{3\sqrt{3}}{2} & -\frac{3}{2} \end{vmatrix} = \left(-\frac{9}{4} - \frac{9(3)}{4}\right)i - (0-0)j + (0-0)k$$

$$= \left(-\frac{36}{4}\right)i - 0j + 0k$$

$= \| \langle -9, 0, 0 \rangle \|$

$\hookrightarrow = \sqrt{9^2} = 9$

$$\kappa = \frac{9}{\left(\sqrt{\frac{9}{4} + \frac{27}{4}}\right)^3}$$

$$= \frac{9}{\left(\sqrt{\frac{36}{4}}\right)^3} = \frac{9}{3^3} = \frac{9}{27} = \frac{1}{3}$$

15. (12 points) Set-up an iterated double integral, in type I format, but do not compute, for the surface area of the surface given parameterically by

$$\mathbf{r}(u,v) = \langle u^2 \,,\, uv \,,\, v^2 \rangle \quad , \quad 0 < u < v < 1 \quad .$$

---

ans. $\int_0^1 \int_u^1 (\sqrt{4v^4 + 16v^2u^2 + 4u^4})\,dv\,du$

$r(u,v) = \langle u^2, uv, v^2 \rangle$ $\qquad 0 < u < v < 1$

$*$ u is on the outside

$\int_0^1 \int_u$

$r_u = \langle 2u, v, 0 \rangle$

$r_v = \langle 0, u, 2v \rangle$

$$\times = \begin{vmatrix} 2u & v & 0 \\ 0 & u & 2v \end{vmatrix}$$

$= (2v^2 - 0)i - (4vu - 0)j + (2u^2 - 0)k$

$= 2v^2 i - 4vu j + 2u^2 k$

$= \langle 2v^2, -4vu, 2u^2 \rangle$

$g(x,y,z) = r(u,v) = u^2 i + uv j + v^2 k$

$|r_u \times r_v| = \sqrt{4v^4 + 16v^2u^2 + 4u^4}$

$|r'(t)| = \sqrt{\ }$

16. (12 points) Let

$$f(x, y, z) = xy^2z^3 \quad ,$$

and let

$$g(x, y, z) = x + y^2 + z^3 \quad .$$

compute the dot-product

$$grad(f) . grad(g) \quad .$$

at the point $(1, 1, 1)$.

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ans. 14

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grad f • grad g

$\nabla f = \langle y^2z^3 , 2yxz^3 , 3z^2y^2x \rangle$ @P= $\langle 1, 2, 3 \rangle$

$\nabla g = \langle 1, 2y, 3z^2 \rangle$ @P = $\langle 1, 2, 3 \rangle$

graf • grad g = $\langle 1, 2, 3 \rangle \cdot \langle 1, 2, 3 \rangle$

$= 1 + 4 + 9 = 14$

$(x+y)(x+y)$

17. (8 points) Decide whether the following limit exists. If it does ,find it. If it does not, explain why it does not exist.

$x^2 + xy + xy + y^2$

$z^2 + 2zw + w^2$

$$\lim_{(x,y,z,w)\to(0,0,0,0)} \frac{(x+y)^2-(z+w)^2}{x+y-z-w}.$$

(x^2+4)

$(x-4)(x+4)$

ans.

$$\lim_{(x,y,z,w)\to(0,0,0,0)} \frac{((x+y)-(z+w))((x+y)+(z+w))}{x+y-z-w}$$

$$\lim_{(x,y,z,w)\to(0,0,0,0)} = \frac{x^2+2xy+y^2-z^2-2zw-w^2}{x+y-z-w} = \frac{0}{0}$$

$$\lim_{(x,y,z,w)\to(0,0,0,0)} =$$