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None.

1. 18.

2. $\int_0^{\frac{1}{2}}\int_{\frac{1}{4}}^{1} f(x,y)\,dx\,dy + \int_{\frac{1}{2}}^{1}\int_{y^2}^{1} f(x,y)\,dx\,dy$

3. $z = -3\sqrt{3}x - 5\sqrt{3}y + (8\sqrt{3}+1)\frac{\pi}{6}$.

4. $3i - 6j + 9k$.

5. $A = \frac{\pi}{3}$ $B = \frac{\pi}{3}$ $C = \frac{\pi}{3}$

6. $-4\sqrt{3}$.

7. $6\cos(2)$.

8. 16π.

9. -15

10. $(\frac{3}{4}, -1)$ is a saddle point.

11. $\frac{8986}{3000}$.

12. $\frac{\sqrt{5}}{6}$.

13. $\int_0^{\frac{\pi}{2}}\int_{\frac{\pi}{2}}^{\pi}\int_0^{4} \rho^6 \sin^4\varphi \cos\varphi \cos^2\theta \sin\theta \, d\rho\, d\theta\, d\varphi$

14. $k = \frac{1}{3}$

15. $\int_0^1\int_0^1 \frac{y^2}{x} \, 2\sqrt{x^2+y^2+y^4/x^2} \, dy\, dx$.

16. 14.

17. exist limit is 0.

signed: Linyang Shan

1. According to

$$\int (\cos(e^{\sin x}) + 5y)\,dy + (\sin(e^{\cos y}) + 11x)\,dx = \iint (11-5)\,dx\,dy = \iint 6\,dx\,dy.$$

$$\iint 6\,dA = 6\times(\text{Area of the surface}) = 6\times(2+\tfrac{1}{[illegible]}) = 15.18$$

2. $\int_{\frac{1}{4}}^{1}\int_0^{\sqrt{x}} f(x,y)\,dy\,dx$ $\{0 \le y \le \sqrt{x}, \frac{1}{4} \le x \le 1\}$.

$\{\cancel{0 \le y \le 1,\ y^2 \le x \le 1}$

$\int_{\frac{1}{4}}^{1}\int_0^{\sqrt{x}} f(x,y)\,dy\,dx = \int_0^{\frac{1}{2}}\int_{\frac{1}{4}}^{1} f(x,y)\,dx\,dy + \int_{\frac{1}{2}}^{1}\int_{y^2}^{1} f(x,y)\,dx\,dy$ $\{0 \le y \le \frac{1}{2}, \frac{1}{4} \le x \le 1\}$ and $\{\frac{1}{2} \le y \le 1, y^2 \le x \le 1\}$

3. $r_x = (1, 0, -3\sqrt{3})$ $r_y = (0, 1, -5\sqrt{3})$

normal vector $= \begin{vmatrix} i & j & k \\ 1 & 0 & -3\sqrt{3} \\ 0 & 1 & -5\sqrt{3} \end{vmatrix} = 3\sqrt{3}i + 5\sqrt{3}j + k$.

the plane is $(z - \frac{\pi}{6}) + 3\sqrt{3}(x - \frac{\pi}{6}) + 5\sqrt{3}(y - \frac{\pi}{6}) = 0$.

$z = -3\sqrt{3}x - 5\sqrt{3}y + (8\sqrt{3}+1)\frac{\pi}{6}$

4. $(\vec{a}+\vec{b}+\vec{c}) \times (2\vec{a} - \vec{b} + 3\vec{c}) = (\vec{a}+\vec{b}+\vec{c}) \times (2\vec{a}) - (\vec{a}+\vec{b}+\vec{c}) \times (\vec{b}) + (\vec{a}+\vec{b}+\vec{c}) \times (3\vec{c})$

$= 2\vec{a}\times\vec{c} - 3\,\vec{a}\times\vec{b} + 4\,\vec{b}\times\vec{c}$

$= (2i+j+2k) - 3(i+j-k) + 4(i-j+k)$

$= 3i - 6j + 9k$.

5. $\vec{AB} = (1, 0, 1)$ $\vec{AC} = (1, 1, 0)$ $\vec{BC} = (0, 1, -1)$.

$\cos A = \frac{\vec{AB}\cdot\vec{AC}}{|\vec{AB}|\cdot|\vec{AC}|} = \frac{1}{2}$ $A = \frac{\pi}{3}$.

$\cos B = \frac{\vec{BA}\cdot\vec{BC}}{|\vec{BA}|\,|\vec{BC}|} = \frac{1}{2}$ $B = \frac{\pi}{3}$.

$\cos C = \frac{\vec{CA}\cdot\vec{CB}}{|\vec{CA}|\cdot|\vec{CB}|} = \frac{1}{2}$ $C = \frac{\pi}{3}$.

6. grad $f(x,y,z) = (3x^2+yz,\ 3y^2+xz,\ 3z^2+xy)$

unit vector $= (\frac{-\sqrt{3}}{3}, \frac{-\sqrt{3}}{3}, \frac{-\sqrt{3}}{3})$.

grad $f(1,1,1) = (4,4,4)$.

directional fraction $= (4,4,4)\cdot(\frac{-\sqrt{3}}{3}, \frac{-\sqrt{3}}{3}, \frac{-\sqrt{3}}{3})$.

$= -4\sqrt{3}$.

7. $\frac{\partial g}{\partial u} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial g}{\partial y}\cdot\frac{\partial y}{\partial u}$. $\quad \frac{\partial x}{\partial u} = e^u\cos v \quad \frac{\partial y}{\partial u} = e^u\sin v$.

$\frac{\partial g}{\partial u} = 6e^u\cos v(e^u\cos v) - 6e^u\sin v(e^u\sin v) = 6e^{2u}\cos(2v)$.

$\frac{\partial g}{\partial u}(0,1) = 6\cos(2)$

8. $\operatorname{div} F = 3+(-2)+5 = 6$.

$\int_S F\,ds = \iiint \operatorname{div}F\,dV = \iiint 6\,dV = 6\times$ (volume of a quater of sphere)

$= 6\times\frac{4}{3}\times\pi\times\frac{2^3}{4} = 16\pi$.

9. $g = 2x+3y$. $\quad \iint F\cdot ds = \iint_D(-P\frac{\partial z}{\partial x} - Q\frac{\partial z}{\partial y} + R)\,dA$

$= \int_0^1\int_0^1(-16x-14y)\,dx\,dy$

using maple $\int_0^1\int_0^1(-16x-14y)\,dx\,dy = -15$

10. $f_x = 4 - \frac{2}{2x+y}$ $f_y = -2y - \frac{1}{2x+y}$ $f_{xx} = \frac{4}{(2x+y)^2}$ $f_{yy} = -2 + \frac{1}{(2x+y)^2}$

$f_{xy} = \frac{2}{(2x+y)^2}$

$\begin{cases} 4 - \frac{2}{2x+y} = 0 \\ -2y - \frac{1}{2x+y} = 0 \end{cases} \Rightarrow \begin{cases} y = -1 \\ x = \frac{3}{4} \end{cases} \Rightarrow \frac{1}{2x+y} = 2$

$D = f_{xx} f_{yy} - (f_{xy})^2 = 32 - 64 = -32 < 0$. $(\frac{3}{4}, -1)$ is a saddle point.

11. $f(1,1,2) = 3$. $f_x = 4x(2x^2+3y^2+z^2)^{-\frac{1}{2}}$ $f_y = 6y(2x^2+3y^2+z^2)^{-\frac{1}{2}}$

$f_z = 2z(2x^2+3y^2+z^2)^{-\frac{1}{2}}$

$f_x(1,1,2) = \frac{4}{3}$ $f_y(1,1,2) = 2$ $f_z(1,1,2) = \frac{4}{3}$

$L = \frac{4}{3}(x-1) + 2(y-1) + \frac{4}{3}(z-2) + 3$.

$= \frac{4}{3}x + 2y + \frac{4}{3}z - 3$.

$L(1.001, 0.999, 2.001) = \frac{8986}{3000}$.

12. The area of this two part of integral is a quarter of circle with $r = 1$, $\theta \in (0, \frac{\pi}{4})$. $(x,y) = (r\cos\theta, r\sin\theta)$.

So the intergral is $\int_0^{\frac{\pi}{4}} \int_0^1 r\cos\theta \cdot r \, dr \, d\theta = \int_0^{\frac{\pi}{4}} \frac{1}{3}\cos\theta \, d\theta$

$= \frac{\sqrt{2}}{6}$

13. $x^2+y^2+z^2=4$ $\quad x<0,\ y>0,\ z>0$

$$\int_0^{\frac{\pi}{2}} \int_{\frac{\pi}{2}}^{\pi} \int_0^{2} \rho^6 \sin^4\varphi \cos\varphi \cos^2\theta \sin\theta \, d\rho \, d\theta \, d\varphi$$

14. $r'(t)=\langle 0,\ 3\cos t,\ -3\sin t\rangle$ $\quad r''(t)=\langle 0,\ -3\sin t,\ -3\cos t\rangle$

$r(\frac{\pi}{3})=\langle 5,\ \frac{3\sqrt{3}}{2},\ \frac{3}{2}\rangle$ $\quad r'(\frac{\pi}{3})=\langle 0,\ \frac{3}{2},\ -\frac{3\sqrt{3}}{2}\rangle$ $\quad r''(\frac{\pi}{3})=\langle 0,\ -\frac{3\sqrt{3}}{2},\ -\frac{3}{2}\rangle$

$$k=\frac{|r'(\frac{\pi}{3})\times r''(\frac{\pi}{3})|}{|r'(\frac{\pi}{3})|^3}=\frac{9}{27}=\frac{1}{3}$$

15. $r_u=\langle 2u,\ v,\ 0\rangle$ $\quad r_v=\langle 0,\ u,\ 2v\rangle$

$$|r_u\times r_v|=\begin{vmatrix} i & j & k \\ 2u & v & 0 \\ 0 & u & 2v \end{vmatrix}=|2v^2 i-2uv j+2u^2 k|=2\sqrt{u^4+v^4+u^2v^2}$$

$$=2\sqrt{x^2+y^2+\frac{y^4}{x^2}}$$

$$\int_0^1\int_0^1 \cancel{\frac{y^2}{x}}\ 2\sqrt{x^2+y^2+y^4/x^2}\ dy\,dx.$$

16. $\operatorname{grad}(f)=(y^2z^3,\ 2xyz^3,\ 3xy^2z^2)$ $\quad \operatorname{grad}(f(1,1,1))=(1,2,3).$

$\operatorname{grad}(g)=(1,\ 2y,\ 3z^2)$ $\quad \operatorname{grad}(g(1,1,1))=(1,2,3).$

$\operatorname{grad}(f(1,1,1))\cdot \operatorname{grad}(g(1,1,1))=1+4+9=14.$

17. $$\frac{(x+y)^2-(z+w)^2}{(x+y)-(z+w)} = x+y+z+w$$

$$\lim_{(x,y,z,w)\to(0,0,0,0)} \frac{(x+y)^2-(z+w)^2}{(x+y)-(z+w)} = \lim_{(x,y,z,w)\to(0,0,0,0)} (x+y+z+w) = 0.$$