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SSC: None.

Answers:

1. 18.

2. $\int_0^1 \int_{y^2}^1 f(x,y)\,dx\,dy$

3.

4. $3i-6j+9k$

5. All are $\frac{\pi}{3}$ radians.

6. $-4\sqrt{3}$.

7. $6\cos 2$.

8. $8\pi$.

9. $-15$.

10 $(\frac{3}{4},-1)$, a saddle point.

11 $\frac{9001}{30000}$.

12 $\frac{\sqrt{2}}{6}$

13 $\int_0^{\frac{\pi}{2}} \int_0^{\pi} \int_0^2 \rho^6 \sin^4\phi \cos\phi \sin\theta \cos^2\theta \, d\rho\, d\theta\, d\phi$

14 $\frac{1}{3}$.

15 $\int_0^1 \int_0^v (2u+v)\times(2v+u)\,du\,dv$

~~$\int_0^1 \int_0^v (2u+v)(2v+u)\,du\,dv$~~

16 14.

17. 0.

Signed: Jiahe Li

I hereby declare that all the work was done by myself. I was allowed to use Maple, calculators (unless told not use), the book and all the material in the web page of this class but not other resources on the Internet.

I only spent 3 hours to do the exam, 30 minutes in checking and double checking the answers.

I also understand I may be subject to random short chat.

1、Ans: 18

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 11 - 5 = 6.$$

$$\int_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA.$$

$$= 6 \cdot \text{Area}$$

$$= 6 \times \left(2\times 1 + \frac{2\times 1}{2}\right)$$

$$= 18.$$

2. Ans. $\cancel{\int_{y^2}^{1}\int_{0}^{1} f(x,y)\,dy\,dx}$ $\int_0^1 \int_{y^2}^{1} f(x,y)\,dx\,dy$

$\cancel{D = \{(x,y) \mid 0 \le x \le \sqrt{x},\ \frac{1}{4} \le y \le 1\}}$

$D = \{(x,y) \mid 0 \le y \le \sqrt{x},\ \frac{1}{4} \le x \le 1\}$

$x \ge y^2$

$\therefore D = \{(x,y) \mid y^2 \le x \le 1,\ 0 \le y \le 1\}$

$\therefore \cancel{\int_{y^2}^{1}\int_{0}^{1} f(x,y)\,dy\,dx}$

$\int_0^1 \int_{y^2}^{1} f(x,y)\,dx\,dy$

$$2\cos\left(\frac{\pi}{3}\right)+4\cos\left(\frac{\pi}{3}\right)+8\cos\left(\frac{\pi}{3}\right)=7.$$

$$-2\frac{\partial z}{\partial y}\sin(x+y)-4\sin(x+z)-8\sin(y+z)\left(1+\frac{\partial z}{\partial y}\right)=0.$$

$$\frac{\partial z}{\partial y}=-\frac{6}{5}$$

$$\frac{\partial z}{\partial x}=$$

4. Ans. $(3i-6j+9k)$

$$(a+b+c)\times(2a-b+3c)$$

$$=-(a\times b)+3(a\times c)+2(b\times a)+3(b\times c)+2(c\times a)-c\times b$$

$$=-i-j+k+6i+3j+6k-2i-2j+2k+3i-3j+3k-4i-2j-4k+i-j+k$$

$$=3i-6j+9k$$

5. Ans. A: $\frac{\pi}{3}$ radians

B: $\frac{\pi}{3}$ radians

C: $\frac{\pi}{3}$ radians

$\overrightarrow{CA} = \langle -1, -1, 0 \rangle$

$\overrightarrow{BA} = \langle -1, 0, -1 \rangle$ $\quad \overrightarrow{AB} = \langle 1, 0, 1 \rangle$ $\quad \overrightarrow{CB} = \langle 0, -1, 1 \rangle$

$\overrightarrow{BC} = \langle 0, 1, -1 \rangle$ $\quad \overrightarrow{AC} = \langle 1, 1, 0 \rangle$

$$\cos C = \frac{0+1+0}{\sqrt{1^2+1^2} \times \sqrt{(-1)^2+1^2}} = \frac{1}{2}$$

$$\cos B = \frac{0+0+1}{\sqrt{1^2+1^2} \times \sqrt{1^2+(-1)^2}} = \frac{1}{2} \qquad \cos A = \frac{1+0+0}{\sqrt{1^2+1^2} \times \sqrt{(-1)^2+(-1)^2}} = \frac{1}{2}$$

$$B = \frac{\pi}{3} \qquad A = \frac{\pi}{3} \qquad C = \frac{\pi}{3}$$

6. Ans. $-4\sqrt{3}$

$$\nabla f = \langle 3x^2 + yz,\ 3y^2 + xz,\ 3z^2 + xy \rangle$$

$$\nabla f(1,1,1) = \langle 4, 4, 4 \rangle$$

$$\sqrt{(-1)^2 + (-1)^2 + (-1)^2} = \sqrt{3}$$

$$u = \frac{1}{\sqrt{3}} \langle -1, -1, -1 \rangle = \langle -\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3} \rangle$$

$$\nabla f \cdot u = -\frac{4}{3}\sqrt{3} - \frac{4}{3}\sqrt{3} - \frac{4}{3}\sqrt{3} = -4\sqrt{3}$$

7. Ans. $\cancel{6e^{2u}\cos 2v}$ $6\cos 2$

$$\frac{\partial x}{\partial u} = e^u \cos v \qquad \frac{\partial y}{\partial u} = e^u \sin v$$

$$\cancel{\frac{\partial x}{\partial v} = -e^u \sin v} \qquad \cancel{\frac{\partial y}{\partial v} = e^u \cos v}$$

$$\frac{\partial g}{\partial x} = 6x. \qquad \frac{\partial g}{\partial y} = -6y.$$

$$\frac{\partial g}{\partial u} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial u}$$

$$= 6e^u \cos v (e^u \cos v) - 6e^u \sin v (e^u \sin v)$$

$$= 6e^{2u} \cos 2v$$

$\because (u, v) = (0, 1)$

$$\frac{\partial g}{\partial u} = 6e^0 \cos 2$$

$$= 6\cos 2$$

8. Ans. $8\pi$

$$\cancel{\iint_S F \cdot dS = \iiint_E \operatorname{div} F \, dV}$$

$$\cancel{\operatorname{div}(F) = 3 - 2 + 5 = 6.}$$

$$\cancel{\int_0^2 \int_{-\sqrt{4-x^2}}^{0} \int_0^{\sqrt{4-x^2-y^2}} 6 \, dz \, dy \, dx.}$$

~~Inner Loop.~~ $\cancel{[6z]_0^{\sqrt{4-x^2-y^2}} = 6\sqrt{4-x^2-y^2}}$

~~Middle Loop.~~

$$\iint_S F \cdot ds = \iiint_E \operatorname{div}(F) \, dV$$

$$\operatorname{div}(F) = 3 - 2 + 5 = 6.$$

It's 6 times the volume of the $\frac{1}{8}$ spherical

$$6 \times \frac{4}{3}\pi \times 2^3 \div 8 = 8\pi$$

9. Ans. $-15$.

~~$div(F) = 1+0+0 = 1$~~

$g = 2x+3y$

$\int_0^1\int_0^1 (-16x-14y)\,dx\,dy = -15$.

$-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R$

$= -6z-6x+y+z$

$= -6x+y-5z$

$= -16x-14y$

10. Ans. $(\frac{3}{4},-1)$ is the saddle point.

$\nabla f = \langle 4-\frac{2}{2x+y}, -2y-\frac{1}{2x+y} \rangle$

when $\nabla f = 0$.

$4-\frac{2}{2x+y}=0$; $-2y-\frac{1}{2x+y}=0$.

$2x+y=\frac{1}{2}$

$-2y-2=0$.

$\begin{cases} y=-1 \\ x=\frac{3}{4} \end{cases}$

$(\frac{3}{4},-1)$ is the critical point.

$f_{xx}=\frac{4}{(2x+y)^2}$ $\quad f_{yy}=-2+\frac{1}{(2x+y)^2}$

$f_{xy}=\frac{2}{(2x+y)^2}$

$D(\frac{3}{4},-1)=\frac{4}{\frac{1}{4}}\times(-2+\frac{1}{\frac{1}{4}})$

$-(\frac{2}{\frac{1}{4}})^2$

$=16\times2-64$

$=-32<0$.

$\therefore$ It's a saddle point.

There's no local maximum or local minimum

$(x,y,z,w)=(0,0,0,0)$

11. Ans. $\frac{9001}{3000}$

$$f(1,1,2)=\sqrt{2+3+2^2}=3.$$

$$\nabla f=\left\langle \frac{2x}{\sqrt{2x^2+3y^2+z^2}},\ \frac{3y}{\sqrt{2x^2+3y^2+z^2}},\ \frac{z}{\sqrt{2x^2+3y^2+z^2}} \right\rangle.$$

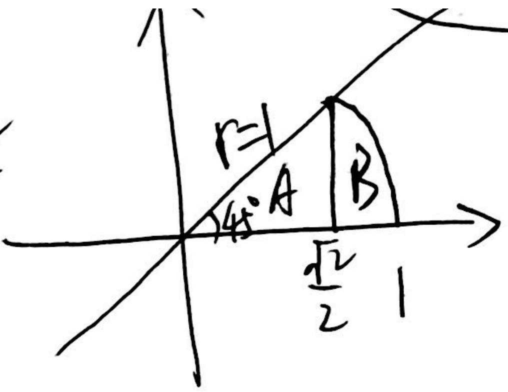
$$\nabla f(1,1,2)=\left\langle \frac{2}{3},\ 1,\ \frac{2}{3} \right\rangle.$$

$$L(x,y,z)=3+\frac{2}{3}(x-1)+(y-1)+\frac{2}{3}(z-2)$$

$$f(1.001,0.999,2.001)\approx 3+\frac{2}{3}\left(\frac{1}{1000}\right)+\left(-\frac{1}{1000}\right)+\frac{2}{3}\left(\frac{1}{1000}\right)$$

$$=\frac{9001}{3000}$$

12. Ans: $\frac{\sqrt{2}}{6}$




It's a $\frac{1}{8}$ circle.

$$\int_0^{\frac{\pi}{4}}\int_0^1 r^2\cos\theta \, dr \, d\theta.$$

Inner Loop: $\left[\frac{r^3}{3}\cos\theta\right]_0^1 = \frac{1}{3}\cos\theta.$

Outer loop: $\left[\frac{1}{3}\sin\theta\right]_0^{\frac{\pi}{4}} = \frac{\sqrt{2}}{6}.$

13. Ans $\int_0^{\frac{\pi}{2}}\int_0^{\pi}\int_0^{2}\rho^6\sin^4\phi\cos\phi\sin\theta\cos^2\theta\, d\rho\, d\theta\, d\phi$

$x^2y z\, dV = \rho^6\sin^4\phi\cos\phi\sin\theta\cos^2\theta\, d\rho\, d\theta\, d\phi$

$x^2+y^2+z^2=4.$

$x\leq 0,\ y\geq 0,\ z\geq 0.$

$\because z\geq 0.\quad \phi\leq\frac{\pi}{2}$

$\because y\geq 0.\quad \theta\leq\pi$

$\therefore E=\{(\rho,\theta,\phi)\,|\,0\leq\rho\leq 2,\ 0\leq\theta\leq\pi,\ 0\leq\phi\leq\frac{\pi}{2}\}$

$$\int_0^{\frac{\pi}{2}}\int_0^{\pi}\int_0^{2}\rho^6\sin^4\phi\cos\phi\sin\theta\cos^2\theta\, d\rho\, d\theta\, d\phi$$

14. Ans $\frac{1}{3}$

$r'(t)=\langle 0, 3\cos t, -3\sin t\rangle.$

$r''(t)=\langle 0, -3\sin t, -3\cos t\rangle.$

$r'(t)\times r''(t)=\langle -9, 0, 0\rangle$

$\sqrt{(-9)^2}=9.$

$|r'(t)|=\sqrt{(3\cos t)^2+(-3\sin t)^2}=3.$

$k(t)=\frac{9}{3^3}=\frac{9}{27}=\frac{1}{3}$

15. Ans. $\int_0^1 \int_0^v (2u+v)\cancel{\times}(2v+u)\,du\,dv$

$$\frac{\partial r}{\partial u} = 2u+v$$

$$\frac{\partial r}{\partial v} = 2v+u.$$

$$\int_0^1 \int_0^v (2u+v)\cancel{\times}(2v+u)\,du\,dv$$

$=$

16. Ans. 14.

$$\nabla f = \langle y^2z^3,\ 2xyz^3,\ 3xy^2z^2 \rangle.$$

$$\nabla f(1,1,1) = \langle 1,2,3 \rangle$$

$$\nabla g = \langle 1, 2y, 3z^2 \rangle$$

$$\nabla g(1,1,1) = \langle 1,2,3 \rangle.$$

$\cancel{\nabla f \cdot \nabla g}$

$$\nabla f\langle 1,1,1\rangle \cdot \nabla g\langle 1,1,1\rangle = \langle 1,2,3\rangle \cdot \langle 1,2,3\rangle$$
$$= 1+4+9$$
$$= \cancel{0}\ 14.$$

17. Ans 0.

Assume $x+y=A$, $z+w=B$.

$$\cancel{\lim_{(x,y,z,w)\to(0,0,0,0)}} \frac{(x+y)^2-(z+w)^2}{x+y-z-w}$$

$$= \frac{A^2-B^2}{A-B}$$

$$= A+B$$

$$= x+y+z+w$$

$$\cancel{\lim} \quad \cancel{(x,y,z,}$$

$$\lim_{(x,y,z,w)\to(0,0,0,0)} x+y+z+w=0.$$

11. Ans $\frac{9001}{3000}$

$$f(1,1,2)=\sqrt{2+3+2^2}=3.$$

$$\nabla f=\left\langle \frac{2x}{\sqrt{2x^2+3y^2+z^2}}, \frac{3y}{\sqrt{2x^2+3y^2+z^2}}, \frac{z}{\sqrt{2x^2+3y^2+z^2}} \right\rangle.$$

$$\nabla f(1,1,2)=\left\langle \frac{2}{3}, 1, \frac{2}{3} \right\rangle.$$

$$L(x,y,z)=3+\frac{2}{3}(x-1)+(y-1)+\frac{2}{3}(z-2)$$

$$f(1.001, 0.999, 2.001)\approx 3+\frac{2}{3}\left(\frac{1}{1000}\right)+\left(-\frac{1}{1000}\right)+\frac{2}{3}\left(\frac{1}{1000}\right)$$

$$=\frac{9001}{3000}$$