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SSC: (circle) None / I / II / I and II

MATH 251 (22,23,24) [Fall 2020], Dr. Z. , Final Exam , Tue., Dec. 15, 2020

Email the completed test, renamed as `finalFirstLast.pdf` to DrZcalc3@gmail.com no later than 3:30pm, (or, in case of conflict, three and half hours after the start).

WRITE YOUR FINAL ANSWERS BELOW

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1. 30
2. $\int_1^1 \int_0^{y^2} f(x,y)\,dx\,dy$
3. $z^{\frac{2}{3}} + \frac{27}{54} - \frac{5x}{6} - \frac{x}{2}$
4. $\langle 1, -4, 7 \rangle$
5. $\frac{\pi}{3}, \frac{\pi}{3}, \frac{\pi}{3}$
6. $-4\sqrt{3}$
7. $6(\cos(1))^2 - 6\cos(1)\sin(1)$
8. 32π
9. -15
10. $(\frac{1}{4}, -1)$ local min
11. 2.999
12. $\frac{\sqrt{2}}{3}$
13. $\int_0^{\pi}\int_0^{2\pi}\int_0^{2} \rho^6 \sin^3\phi \cos\theta \cos\phi \sin\theta \, d\rho\, d\theta\, d\phi$
14. $\frac{1}{3}$
15. $\int_0^1 \int_0^1 v^2 + 4uv + 2u^2$
16. 1°
17. 0.0001

Sign the following declaration:

I Hereby declare that all the work was done by myself. I was allowed to use Maple (unless specifically told not to), calculators, the book, and all the material in the web-page of this class but **not** other resources on the internet.

I only spent (at most) 3 hours on doing the exam. The last 30 minutes were spent in checking and double-checking the answers.

I also understand that I may be subject to a random short chat to verify that I actually did it all by myself.

Signed: Jgeb Race

Possibly useful facts from school Geometry (that you are welcome to use) : (i) The area of a circle radius r is πr^2. (ii) The circumference of a circle radius r is $2\pi r$ (iii) The parametric equation of an ellipse with axes a b and parallel to the x and y axes respectively is $x = a\cos\theta$, $y = b\cos\theta$, $0 < \theta < 2\pi$. (iv) The area of an ellipse with axes a and b is πab (v) The volume and surface area of a sphere radius R are $\frac{4}{3}\pi R^3$ and $4\pi R^2$ respectively (vi) The volume of a cone is the area of the base times the height over 3. (vii) The volume of a pyramid is the area of the base times the height over 3. (viii) The area of a triangle is base times height over 2.

Formula that you may (or may not) need

If the surface S is given in **explicit** notation $z = g(x, y)$, above the region of the xy-plane , D, then

$$\int\int_S \mathbf{F} \cdot d\mathbf{S} =$$

$$\int\int_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R \right) dA \ .$$

1. (12 pts.) **Without using Maple (or any software)** Compute the vector-field line integral

$$\int_C (\cos(e^{\sin x}) + 5y)\, dx + (\sin(e^{\cos y}) + 11x)\, dy \quad ,$$

over the path consisiting of the five line segments (in that order)

$$(1,0) \to (-1,0) \to (-1,1) \to (0,2) \to (1,1) \to (1,0) \quad .$$

Explain!

ans. 30

$$\iint (11-5y)\, dx\, dy$$

$$\iint 6\, dx\, dy$$

$$6 \cdot 5 = 30$$

We find the integral using green's theorm since it will be the same for each of the five paths we do $5 \cdot 6 = 30$

2. (12 points) Change the order of integration

$$\int_{\frac{1}{4}}^{1}\int_{0}^{\sqrt{x}} f(x,y)\,dy\,dx \;.$$

---

ans. $$\int_{\frac{1}{2}}^{1}\int_{0}^{y^2} f(x,y)\,dx\,dy$$

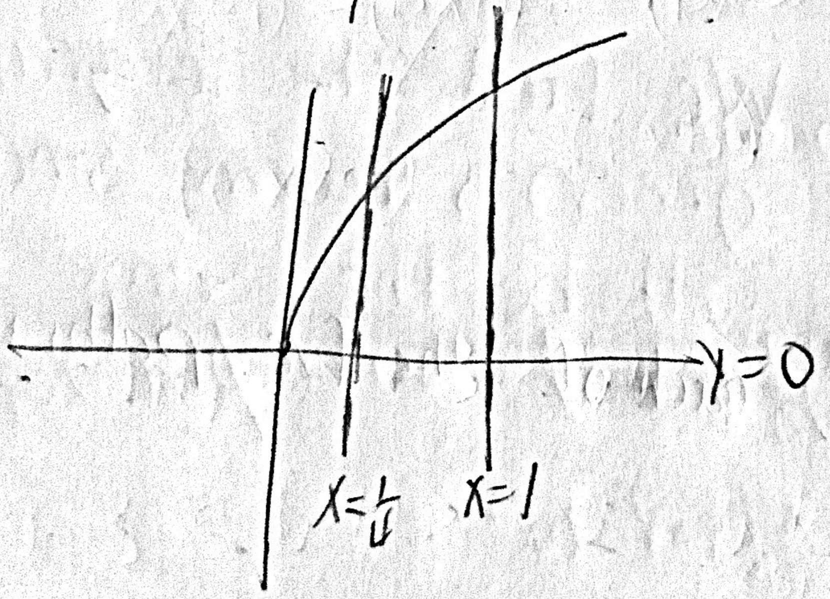
$$\int_{\frac{1}{4}}^{1}\int_{0}^{\sqrt{x}} f(x,y)\,dy\,dx$$

$x=y^2$

$y=\sqrt{x}$ $x=1$

$x=\frac{1}{4}$

$y=0$



3. (12 points) Find the equation of the tangent plane at the point $(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6})$ to the surface given implicitly by

$$2\cos(x+y) + 4\cos(x+z) + 8\cos(y+z) = 7$$

Express you answer in **explicit** form, i.e in the format $z = ax + by + c$.

ans. $z = + \frac{2\pi}{54} - \frac{5y}{6} - \frac{x}{2}$

$f_x = -2\sin(x+y) - 4\sin(x+z)$

$z - \pi$

$f_y = -2\sin(x+y) - 8\sin(x+y)$

$\frac{\pi}{6} + \frac{2\pi}{9}$

$\frac{9\pi}{54} + \frac{12\pi}{54} = \frac{2\pi}{54}$

$f_z = -4\sin(x+z) - 8\sin(y+z)$

$-4\sin(\frac{\pi}{3}) - 8\sin(\frac{\pi}{3})$

$-4\frac{\sqrt{3}}{2} - 8\frac{\sqrt{3}}{2}$

$-2\sin(\frac{2\pi}{6}) - 4\sin(\frac{2\pi}{6})$

$-2\sin(\frac{\pi}{3}) - 8\sin(\frac{\pi}{3})$ $-2\sqrt{3}$ $-4\sqrt{3}$

$-2\frac{\sqrt{3}}{2} - 4\frac{\sqrt{3}}{2}$ $-\sqrt{3} - 2\sqrt{3}$ $-\sqrt{3} - 4\sqrt{3}$

$\langle -3\sqrt{3}, -5\sqrt{3}, -6\sqrt{3} \rangle$

$-3\sqrt{3}(x - \frac{\pi}{6}) - 5\sqrt{3}(y - \frac{\pi}{6}) - 6\sqrt{3}(z - \frac{\pi}{6}) = 0$

$-3\sqrt{3}x + \frac{\sqrt{3}\pi}{2} - 5\sqrt{3}y + \frac{5\sqrt{3}\pi}{6} - 6\sqrt{3}z + \sqrt{3}\pi = 0$

$\frac{-3\sqrt{3}x}{6\sqrt{3}} - \frac{5\sqrt{3}y}{6\sqrt{3}} + \frac{\frac{8\sqrt{3}\pi}{6}}{6\sqrt{3}} + \frac{\sqrt{3}\pi}{6\sqrt{3}} = \frac{6\sqrt{3}z}{6\sqrt{3}}$

$\frac{8\pi}{36} - \frac{4\pi}{18} = \frac{2\pi}{9}$

$-\frac{x}{2} - \frac{5}{6}y + \frac{2\pi}{9} + \frac{\pi}{6} = z$

4. (16 points) Let **a**, **b**, **c** be three vectors such that

$$\mathbf{a}\times\mathbf{b}=\mathbf{i}+\mathbf{j}-\mathbf{k} \quad , \quad \mathbf{b}\times\mathbf{c}=\mathbf{i}-\mathbf{j}+\mathbf{k} \quad , \quad \mathbf{a}\times\mathbf{c}=2\mathbf{i}+\mathbf{j}+2\mathbf{k} \ .$$

What is

$$(\mathbf{a}+\mathbf{b}+\mathbf{c})\times(2\mathbf{a}-\mathbf{b}+3\mathbf{c}) \qquad ?$$

---

ans. $\langle 1,-4,7\rangle$

$a\times b = \langle 1,1,-1\rangle$

$b\times c = \langle 1,-1,1\rangle$

$a\times c = \langle 2,1,1\rangle$

$\cancel{(a\times 2a)} - (a\times b) + (a\times 3c) +$

$(b\times 2a) - \cancel{(b\times b)} + (b\times 3c)$

$+ (c\times 2a) - (c\times b) + \cancel{(c\times 3c)}$

$-\langle 1,1,-1\rangle + 3\langle 2,1,2\rangle + 2\langle -1,-1,1\rangle$

$+ 3\langle 1,-1,1\rangle + 2\langle -2,-1,-2\rangle + \langle -1,1,-1\rangle$

$\langle 5,2,7\rangle \qquad \langle 3,0,9\rangle$

$\langle -1,-1,1\rangle + \langle 6,3,6\rangle + \langle -2,-2,2\rangle$

$6 \quad \langle 4,-3,12\rangle \quad \langle 2,-5,8\rangle \quad \langle 1,-4,7\rangle$

$\langle 3,-3,3\rangle + \langle -4,-2,-4\rangle + \langle -1,1,-1\rangle$

5. (12 points) Find the three angles of the triangle ABC where

$$A = (0,0,0) \quad , \quad B = (1,0,1) \quad , \quad C = (1,1,0) \quad .$$

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ans. The angle at A is: $\frac{\pi}{3}$ radians ;

The angle at B is: $\frac{\pi}{3}$ radians ;

The angle at C is: $\frac{\pi}{3}$ radians ;

A:

$\langle 1, 0, 1 \rangle$ $\quad \dfrac{1+0+0}{\sqrt{2}\sqrt{2}} = \dfrac{1}{2} = \cos^{-1}\left(\frac{1}{2}\right)$

$\langle 1, 1, 0 \rangle$

$\sqrt{1+1}$ $\qquad \frac{\pi}{3}$

B:

$\langle -1, 0, -1 \rangle$ $\quad \dfrac{1+0+0}{\sqrt{2}\sqrt{2}} = \frac{1}{2} = \cos^{-1}\left(\frac{1}{2}\right)$

$\langle -1, -1, 0 \rangle$

C:

$\langle -1, -1, 0 \rangle$ $\quad \dfrac{0+1+0}{\sqrt{2}\sqrt{2}} = \frac{1}{\sqrt{2}\,2} = \cos^{-1}\left(\frac{1}{2}\right)$

$\langle 0, -1, 1 \rangle$ $\qquad \frac{\pi}{3}$

$\langle -5, -1, -5 \rangle$

$\langle -2, -4, -2 \rangle$ $\langle -4, -6, 0 \rangle$ $\langle 2, -3, 6 \rangle$ $\langle 1, -4, 7 \rangle$

6. (12 points) Find the directional derivative of

$$f(x, y, z) = x^3 + y^3 + z^3 + xyz \quad ,$$

at the point $(1, 1, 1)$ in a direction pointing to the point $(-1, -1, -1)$.

ans. $-4\sqrt{3}$

$(-1-1, -1-1, -1-1)$

$\langle -2, -2, -2 \rangle$

$\langle 3x^2 + yz, 3y^2 + xz, 3z^2 + xy \rangle$

$\langle 4, 4, 4 \rangle \cdot \langle \frac{-2}{\sqrt{12}}, \frac{-2}{\sqrt{12}}, \frac{-2}{\sqrt{12}} \rangle$ $\quad \sqrt{2^2 + 2^2 + 2^2}$

$\sqrt{4+4+4}$

$\sqrt{12}$

$\frac{-8}{\sqrt{12}} \ \frac{-8}{\sqrt{12}} \ \frac{-8}{\sqrt{12}} = \frac{-24}{\sqrt{12}} = \frac{-24}{2\sqrt{3}}$

$\sqrt{4}\sqrt{3}$

$= \frac{-12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$= -4\sqrt{3}$

7. (12 points) Using the Chain Rule (no credit for other methods), find

$$\frac{\partial g}{\partial u}$$

at $(u, v) = (0, 1)$, where

$$g(x, y) = 3x^2 - 3y^2 \quad ,$$

and

$$x = e^u \cos v \quad , \quad y = e^u \sin v \quad .$$

ans. $6((\cos(1))^2 - 6\cos(1)\sin(1))$

$$\frac{\partial g}{\partial u} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial v}$$

$$(6x)(e^u \cos v) + (-6y)(\cos v e^u)$$

$$(6x)(e^0 \cos(1)) + (-6y)(\cos(1) e^0)$$

$$6x \cos(1) - 6y \cos(1)$$

$$6\cos(1)x - 6(\cos(1))y$$

$$6\cos(1)(e^u \cos v) - 6(\cos(1))(e^0 \sin v)$$

$$6\cos(1)(\cos(1)) - 6\cos(1)(e^0 \sin(1))$$

$$6\cos^2(1) - 6\cos(1)\sin(1)$$

8. (12 points) Without using Maple (or any other software), compute the vector-f[ield] surface integral $\int_S \mathbf{F}.d\mathbf{S}$ if

$$\mathbf{F} = \langle 3x + \cos(y^3 + yz), -2y + e^{x+z^2}, 5z + \sin(xy^3 + e^x) \rangle$$,

and S is the closed surface in 3D space bounding the region

$$\{(x, y, z) : x^2 + y^2 + z^2 < 4 \ \ and \ \ x > 0 \ \ and \ \ y < 0 \ \ and \ \ z > 0\}$$.

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ans. $\frac{32\pi}{3}$

$\operatorname{div}(F) = \frac{\partial}{\partial x}(3x + \cos(y^3 + yz))$

$+ \frac{\partial}{\partial y}(-2y + e^{x+z^2})$

$+ \frac{\partial}{\partial z}(5z + \sin(xy^3 + e^x))$

$= 3 - 2 + 5 = 1 + 5 = 6$

$\int\int\int 6 \, dx\,dy\,dz$

$\int_0^{\pi}\int_0^{\pi}\int_0^2 6\rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

$\frac{\rho^3}{3}\Big|_0^2$

$\frac{8\pi}{3} - \cos\phi\Big|_0^{\pi}$ $\qquad \frac{16\pi}{3}$ $\qquad \int_0^{\pi}\frac{8}{3}\,d\theta$ $\qquad \frac{8\pi}{3}\sin\phi$

$1+1$

$3\overline{)96}$ 32

$16 \times 6 = 96$

10

9. (12 points) Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if

$$\mathbf{F} = \langle 3z\,,\, 2x\,,\, y+z \rangle \quad ,$$

and S is the oriented surface

$$z = 2x + 3y \quad , \quad 0 < x < 1, \quad 0 < y < 1 \quad ,$$

with **upward pointing** normal.

ans. -15

$$\int\int (-3z)(2) - (2x)(3) + y + z)\,dx\,dx$$

$$\int_0^1\int_0^1 (-6z - 6x + y + z)\,dx\,dx$$

$$\int_0^1\int_0^1 (-6x + y - 5z)\,dx\,dx$$

$$\int_0^1\int_0^1 (-6x + y - 5(2x + 3x)))\,dx\,dx$$

$-16x - 7y^2$

$$\int_0^1\int_0^1 (-6x + y - 10x - 15x)\,dx\,dx$$

$-16x - 7$

$-8x^2 - 7x$

$$\int_0^1\int_0^1 (-16x - 14y)\,dy\,dx$$

$-19yx + \frac{y^2}{2}\Big|_0^1$

$-\frac{19}{2} + \frac{1}{2} = -\frac{18}{2} = -9$

$$\int_0^1 -16x - 7\,dx$$

$-8x^2 - 7x$

$-8 - 7$

$= -15x + \frac{1}{2}$

10. (12 points) Without using Maple or software, find the critical point(s) of

$$f(x,y) = 4x - y^2 - \ln(2x+y) \quad ,$$

and decide for each whether it is a local maximum, local minimum, or saddle point. Explain.

ans. $(\frac{1}{4}, -1)$ local min since $f_{xx} > 0$ and $D > 0$

$(\frac{1}{4}, -1)$ $\quad f_x(x,y) = 4 - \frac{2}{2x+y} \qquad 4 - \frac{2}{2(\frac{1}{4})+y} \qquad 4 - \frac{2}{\frac{1}{2}+y} \qquad -\frac{2}{2x+y} = -4 \qquad \frac{2}{2x+y} = 4$

$f_{xx}(x,y) = -\frac{-(2)(2)}{(2x+y)^2} = \frac{4}{(2x+y)^2} \qquad \frac{4}{\frac{1}{4}}$

$(16)(2) - 16$

$x = \frac{1}{4}$ $\qquad -\frac{1}{2} = 2x+y$

$y = -1$ $\quad f_y(x,y) = -2y - \frac{1}{2x+y} \qquad -y = 2x - \frac{1}{2} \qquad y = \frac{1}{2} - 2x$

$-2y = \frac{1}{2x+y} \qquad -\frac{1}{2x+y} = 2y \qquad x = \frac{1}{4}$

$-\frac{1}{2} = \frac{1}{2x+\frac{1}{4}} \qquad -1 = 2y(2x+y)$

$f_{xy}(x,y) = -2 - \frac{-1}{(2x+y)^2}$

$2 - \frac{1}{2(\frac{1}{4})-1} \qquad = -2 + \frac{1}{(2x+y)^2} \qquad -\frac{1}{\frac{1}{2}+y} = 2y$

$2 - \frac{1}{\frac{1}{2}-1} \qquad -1 = 2y(\frac{1}{2}+y)$

$-1 = y + 2y^2$

$f_{xy}(x,y) = -\frac{-1}{(2x+y)^2} = \frac{1}{(2x+y)^2} \qquad 2y^2 + y$

$\frac{1}{\frac{1}{2}-1} \qquad \frac{1}{-\frac{1}{4}}$

11. (12 points) Without using Maple or software, using a **Linearization** around the point $(1,1,2)$, approximate $f(1.001, 0.999, 2.001)$ if

$$f(x,y,z) = \sqrt{2x^2+3y^2+z^2} \quad .$$

ans. 2.999

$$f_x(1,1,2) = \frac{4x}{2\sqrt{2x^2+3y^2+z^2}} = \frac{4}{2\sqrt{2+3+4}} (2x^2+3y^2+z^2)^{\frac{1}{2}}$$

$$= \frac{4}{2\sqrt{9}} = \frac{4}{6} = \frac{2}{3}$$

$$f_y(1,1,2) = \frac{6y}{2\sqrt{2x^2+3y^2+z^2}} = \frac{6}{2\sqrt{2+3+4}} = \frac{3}{\sqrt{9}} = 1$$

$$f_z(1,1,2) = \frac{2z}{2\sqrt{2x^2+3y^2+z^2}} = \frac{z}{\sqrt{2+3+4}} = \frac{2}{3}$$

$$\sqrt{2+3+4} = \sqrt{9} = 3$$

$$3 + \frac{2}{3}(1.001-1) + 1(0.999-1) + \frac{2}{3}(2-2.001)$$

$$3 + \frac{2}{3}(.001) + 1(-.001) + \frac{2}{3}(-.001)$$

$$3 - .001 = 2.999$$

12. (12 points) Without using Maple (or any other software) and by using polar coordinates (no credit for doing it directly) find

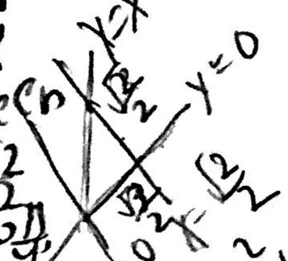
$$\int_0^{\frac{\sqrt{2}}{2}} \int_0^{x} x\,dy\,dx + \int_{\frac{\sqrt{2}}{2}}^{1} \int_0^{\sqrt{1-x^2}} x\,dy\,dx \quad .$$

Explain!

ans. $\frac{\sqrt{2}}{3}$

We turn x to $r\cos\theta$ and multiplied with r since x is between 0 and $\frac{\sqrt{2}}{2}$ it is 0 to $\frac{\pi}{4}$.

$\int_0^{\frac{\pi}{4}} \int_0^{1} r^2\cos\theta\, dr\, d\theta \rightarrow \frac{\sqrt{2}}{6}$



$\frac{r^3}{3}\Big|_0^1$

$\frac{1}{3}$

$\frac{\sqrt{2}}{6} + \frac{\sqrt{2}}{6} = \frac{2\sqrt{2}}{6}$

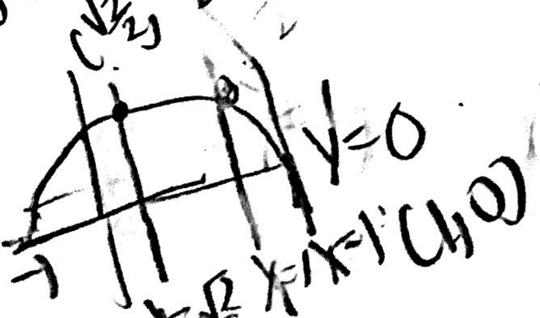
$\frac{\sqrt{2}}{3}$

$\sqrt{1-2/4}$ $\sqrt{2/4}$ $\sqrt{2}/2$ $(\frac{\sqrt{2}}{2})^2 + (\frac{\sqrt{2}}{2})^2$ $\frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1$

$\frac{1}{3}\int_0^{\frac{\pi}{4}} \cos\theta\, d\theta \rightarrow \frac{\sqrt{2}}{6}$

$\sin\theta\Big|_0^{\frac{\pi}{4}}$

This is the same for the other integral where $1 = \sqrt{1-x^2}$ and $\frac{\sqrt{2}}{2} = \sqrt{1-x^2}$ meet up between 0 and $\frac{\pi}{4}$



$\frac{\sqrt{2}}{2}\left(\frac{1}{3}\right)\frac{\sqrt{2}}{2} - 0 = \frac{\sqrt{2}}{2}$

$\int_0^{\frac{\pi}{4}} \int_0^{1} r^2\cos\theta\, dr\, d\theta$

$\frac{1}{3}\int_0^{\frac{\pi}{4}} \cos\theta\, d\theta$

$\sin\theta\Big|_0^{\frac{\pi}{8}}$

$\frac{r^3}{3}\Big|_0^1$ $\frac{1}{3}$ $\frac{\sqrt{2}}{2}$ $\frac{\sqrt{2}}{6}$ $=0$

13. (12 points) Convert the triple iterated integral

$$\int_0^{2} \int_0^{\sqrt{4-z^2}} \int_{-\sqrt{4-z^2-y^2}}^{0} x^2\, y\, z\, dx\, dy\, dz$$

to spherical coordinates. Do not evaluate.

ans. $\int_0^{\pi} \int_0^{2\pi} \int_0^{2} \rho^6 \sin^3\phi \cos\theta \cos\phi \sin\theta \, d\rho\, d\theta\, d\phi$

$$\int_0^{2} \int_0^{\sqrt{4-y^2}} \int_{-\sqrt{4-z^2-y^2}}^{0} x^2 y z \, dx\, dy\, dz$$

$$\iiint (\rho^2 \sin\phi)(\rho \sin\phi \cos\theta)^2 (\rho \sin\phi \sin\theta)$$

$$(\rho \cos\phi)\, d\rho\, d\theta\, d\phi \qquad \rho^2 \cdot \rho^2 \cdot \rho \cdot \rho$$

$$\rho^4 \cdot \rho \cdot \rho$$

$$\int_0^{\pi} \int_0^{2\pi} \int_0^{2} \rho^6 \sin^3\phi \cos\theta \cos\phi \sin\theta \, d\rho\, d\theta\, d\phi$$

14. (12 points) Find the curvature of the curve

$$\mathbf{r}(t) = \langle 5, 3\sin t, 3\cos t \rangle$$

at the point where $t = \frac{\pi}{3}$.

ans. $\frac{1}{3}$

$r'(t) = \langle 0, 3\cos t, -3\sin t \rangle$

$r''(t) = \langle 0, -3\sin t, -3\cos t \rangle$

$r'(\frac{\pi}{3}) = \langle 0, \frac{3}{2}, \frac{-3\sqrt{3}}{2} \rangle$

$3\sqrt{3}\ 3\sqrt{3}$

$r''(\frac{\pi}{3}) = \langle 0, \frac{-3\sqrt{3}}{2}, \frac{-3}{2} \rangle$

$-\frac{9}{4} - \frac{27}{4} = \frac{-36}{4} = -9$

$-9i - 0j - 0k$

$\sqrt{9^2} = 9$ $\qquad \frac{9}{3^3} = \frac{9}{27} = \frac{1}{3}$

$\sqrt{0^2 + \frac{9}{4} + \frac{27}{4}}$ $\sqrt{\frac{36}{4}}$ $\sqrt{9}$ 3

15. (12 points) Set-up an iterated double integral, in type I format, but do not compute, for the surface area of the surface given parameterically by

$$\mathbf{r}(u,v) = \langle u^2\,,\, uv\,,\, v^2\rangle \quad , \quad 0 < u < v < 1\ .$$

ans. $\int_0^1 \int_0^1 v^2 + 4uv + 2u^2$

$\qquad i \quad j \quad k$

$r_u = \langle 2u, v, 0\rangle$

$r_v \langle 0, u, 2v\rangle$

$v^2 + 4uv + 2u^2$

$\int_0^1 \int_0^1$

16. (12 points) Let

$$f(x, y, z) = xy^2z^3 \quad ,$$

and let

$$g(x, y, z) = x + y^2 + z^3 \quad .$$

compute the dot-product

$$grad(f) \, . \, grad(g) \quad .$$

at the point (1, 1, 1).

---

ans. 14

$grad(f) = \langle y^2z^3, 2yxz^3, 3z^2xy^2 \rangle$

$grad(g) = \langle 1, 2y, 3z^2 \rangle$

$\langle 1, 2, 3 \rangle \qquad \langle 1, 2, 3 \rangle$

$\langle 1, 2, 3 \rangle \cdot \langle 1, 2, 3 \rangle$

$1 + 4 + 9 = 14$

17. (8 points) Decide whether the following limit exists. If it does ,find it. If it does not, explain why it does not exist.

$$\lim_{(x,y,z,w)\to(0,0,0,0)} \frac{(x+y)^2-(z+w)^2}{x+y-z-w}.$$

ans. I 0.0001 ~~exist~~

$$\lim_{(x,y,z,w)\to(0,0,0,0)} \frac{(x+y)^2-(z+w)^2}{x+y-z-w}$$

$$= \frac{0^2-0^2}{0+0-0-0} = \frac{0}{0}$$

$$\lim_{(x,y,z,w)\to(0,0,0,0.0001)} = \frac{0^2-(0.001)^2}{0+0-0-0.0001}$$

$$= \frac{-(0.001)^2}{-0.0001}$$

$$= 0.0001$$

$$\lim_{(x,y,z,w)\to(0,0.0001,0,0)} = \frac{(0.0001)^2-0^2}{0.0001-0}$$

$$= 0.0001$$

As it approaches to 0.0001 the value equal to 1 meaning the limit probably does exist