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SSC: (circle) None / I / II / I and II

MATH 251 (22,23,24) [Fall 2020], Dr. Z. , Final Exam , Tue., Dec. 15, 2020

Email the completed test, renamed as `finalFirstLast.pdf` to DrZcalc3@gmail.com no later than 3:30pm, (or, in case of conflict, three and half hours after the start).

WRITE YOUR FINAL ANSWERS BELOW

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1. 18
2. $\int_0^{1/2}\int_{1/4}^{1} f(x,y)\,dx\,dy + \int_{1/2}^{1}\int_{y^2}^{1} f(x,y)\,dx\,dy$
3. $z = -\frac{1}{2}x - \frac{5}{6}y + \frac{7\pi}{18}$
4. $\langle 3, -6, 9 \rangle$
5. $\theta_A = \frac{\pi}{3}$; $\theta_B = \frac{\pi}{3}$; $\theta_C = \frac{\pi}{3}$
6. $-4\sqrt{3}$
7. $6\cos^2(1) - 6\sin^2(1)$
8. 8π
9. -15
10. $\left(\frac{3}{4}, -1\right)$ is a saddle point
11. $\frac{9001}{3000}$
12. $\frac{1}{3\sqrt{2}}$
13. $\int_0^2\int_0^{\frac{\pi}{2}}\int_{\frac{\pi}{2}}^{\pi} \rho^6 \sin^4\phi \cos^2\theta \cos\phi \sin\theta \, d\theta\, d\phi\, d\rho$
14. $\frac{1}{3}$
15. $\int_0^1\int_0^v 2v\sqrt{v^2+4u^2}\, du\, dv$
16. 14
17. 0

Sign the following declaration:

I Hereby declare that all the work was done by myself. I was allowed to use Maple (unless specifically told not to), calculators, the book, and all the material in the web-page of this class but **not** other resources on the internet.

I only spent (at most) 3 hours on doing the exam. The last 30 minutes were spent in checking and double-checking the answers.

I also understand that I may be subject to a random short chat to verify that I actually did it all by myself.

Signed:

Possibly useful facts from school Geometry (that you are welcome to use) : (i) The area of a circle radius r is πr^2. (ii) The circumference of a circle radius r is $2\pi r$ (iii) The parametric equation of an ellipse with axes a b and parallel to the x and y axes respectively is $x = a\cos\theta$, $y = b\cos\theta$, $0 < \theta < 2\pi$. (iv) The area of an ellipse with axes a and b is πab (v) The volume and surface area of a sphere radius R are $\frac{4}{3}\pi R^3$ and $4\pi R^2$ respectively (vi) The volume of a cone is the area of the base times the height over 3. (vii) The volume of a pyramid is the area of the base times the height over 3. (viii) The area of a triangle is base times height over 2.

Formula that you may (or may not) need

If the surface S is given in **explicit** notation $z = g(x, y)$, above the region of the xy-plane , D, then

$$\int\int_S \mathbf{F} \cdot d\mathbf{S} =$$

$$\int\int_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R \right) dA \quad .$$

1. (12 pts.) **Without using Maple (or any software)** Compute the **vector-field line integral**

$$\int_C (cos\,(e^{\sin x}) + 5y)\,dx \;+\; (sin\,(e^{\cos y}) \;+\; 11x\,)\,dy \quad ,$$

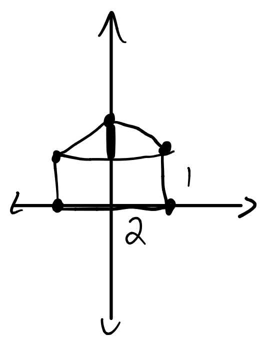
over the path consisiting of the five line segments (in that order)

$$(1,0) \to (-1,0) \to (-1,1) \to (0,2) \to (1,1) \to (1,0) \quad .$$

Explain!

---

ans. 18



$\frac{1}{2}bh$

→ Greene's Theorem

→ $P = \cos(e^{\sin x}) + 5y$; $P_y = 5$

→ $Q = \sin(e^{\cos y}) + 11x$; $Q_x = 11$

→ $\iint_D 6\,dx\,dy$

→ Since we have a constant, multiply it by area of pentagon:

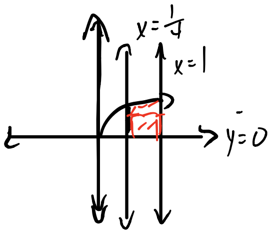
→ $6\left(\frac{1}{2} + \frac{1}{2} + 2\right) = 6(3) = \boxed{18}$

2. (12 points) Change the order of integration

$$\int_{\frac{1}{4}}^{1}\int_{0}^{\sqrt{x}} f(x,y)\,\overset{dy\,dx}{\cancel{dx\,dy}} \qquad .$$

---

ans. $\displaystyle\int_0^{\frac{1}{2}}\int_{\frac{1}{4}}^{1} f(x,y)\,dx\,dy + \int_{\frac{1}{2}}^{1}\int_{y^2}^{1} f(x,y)\,dx\,dy$

→ 

→ Must be split into two integrals

→ $$\boxed{\int_0^{\frac{1}{2}}\int_{\frac{1}{4}}^{1} f(x,y)\,dx\,dy + \int_{\frac{1}{2}}^{1}\int_{y^2}^{1} f(x,y)\,dx\,dy}$$

3. (12 points) Find the equation of the tangent plane at the point $(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6})$ to the surface given implicitly by

$$2\cos(x+y) + 4\cos(x+z) + 8\cos(y+z) = 7 \qquad .$$

Express you answer in **explicit** form, i.e in the format $z = ax + by + c$.

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ans. $z = -\frac{1}{2}x - \frac{5}{6}y + \frac{7\pi}{18}$

$\rightarrow f(x,y,z) = 2\cos(x+y) + 4\cos(x+z) + 8\cos(y+z) - 7$

$\rightarrow f_x = -4\sin(x+z) - 2\sin(x+y)$

$\rightarrow f_y = -8\sin(y+z) - 2\sin(x+y)$

$\rightarrow f_z = -8\sin(y+z) - 4\sin(x+z)$

$\rightarrow f_x\left(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}\right) = -4\sin\left(\frac{\pi}{3}\right) - 2\sin\left(\frac{\pi}{3}\right) = -4\cdot\frac{\sqrt{3}}{2} - 2\cdot\frac{\sqrt{3}}{2} = -3\sqrt{3}$

$\rightarrow f_y\left(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}\right) = -8\sin\left(\frac{\pi}{3}\right) - 2\sin\left(\frac{\pi}{3}\right) = -8\cdot\frac{\sqrt{3}}{2} - 2\cdot\frac{\sqrt{3}}{2} = -5\sqrt{3}$

$\rightarrow f_z\left(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}\right) = -8\sin\left(\frac{\pi}{3}\right) - 4\sin\left(\frac{\pi}{3}\right) = -8\cdot\frac{\sqrt{3}}{2} - 4\cdot\frac{\sqrt{3}}{2} = -6\sqrt{3}$

$\rightarrow -3\cancel{\sqrt{3}}\left(x - \frac{\pi}{6}\right) - 5\cancel{\sqrt{3}}\left(y - \frac{\pi}{6}\right) - 6\cancel{\sqrt{3}}\left(z - \frac{\pi}{6}\right) = 0$

$\rightarrow -3\left(x - \frac{\pi}{6}\right) - 5\left(y - \frac{\pi}{6}\right) - 6\left(z - \frac{\pi}{6}\right) = 0$

$\rightarrow \boxed{z = -\frac{1}{2}x - \frac{5}{6}y + \frac{7\pi}{18}}$

4. (16 points) Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three vectors such that

$$\mathbf{a} \times \mathbf{b} = \mathbf{i} + \mathbf{j} - \mathbf{k} \quad , \quad \mathbf{b} \times \mathbf{c} = \mathbf{i} - \mathbf{j} + \mathbf{k} \quad , \quad \mathbf{a} \times \mathbf{c} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} \quad .$$

What is

$$(\mathbf{a} + \mathbf{b} + \mathbf{c}) \times (2\mathbf{a} - \mathbf{b} + 3\mathbf{c}) \qquad ?$$

---

ans. $\langle 3, -6, 9 \rangle$

$\rightarrow \cancel{2(a \times a)}^{0} - a \times b + 3(a \times c) + 2(b \times a) - \cancel{b \times b}^{0} + 3(b \times c) + 2(c \times a)$
$- c \times b + \cancel{3(c \times c)}^{0}$

$\rightarrow \langle -1, -1, 1 \rangle + 3\langle 2, 1, 2 \rangle + 2\langle -1, -1, 1 \rangle + 3\langle 1, -1, 1 \rangle + 2\langle -2, -1, -2 \rangle$
$- \langle -1, 1, -1 \rangle$

$\rightarrow \langle -1, -1, 1 \rangle + \langle 6, 3, 6 \rangle + \langle -2, -2, 2 \rangle + \langle 3, -3, 3 \rangle + \langle -4, -2, -4 \rangle$
$+ \langle 1, -1, 1 \rangle$

$\rightarrow \boxed{\langle 3, -6, 9 \rangle}$

5. (12 points) Find the three angles of the triangle ABC where

$$A = (0,0,0) \quad , \quad B = (1,0,1) \quad , \quad C = (1,1,0) \quad .$$

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ans. The angle at A is: $\frac{\pi}{3}$ radians ;

The angle at B is: $\frac{\pi}{3}$ radians ;

The angle at C is: $\frac{\pi}{3}$ radians ;

$\to \cos\theta = \dfrac{u \cdot v}{|u| \cdot |v|}$

$\to \overline{AB} = B - A = (1,0,1) - (0,0,0) = \langle 1,0,1 \rangle$

$\to \overline{AC} = C - A = (1,1,0) - (0,0,0) = \langle 1,1,0 \rangle$

$\to \overline{BC} = C - B = (1,1,0) - (1,0,1) = \langle 0,1,-1 \rangle$

$\to \overline{BA} = -\overline{AB} = \langle -1,0,-1 \rangle$

$\to$ The angle at A:

$$\cos\theta = \frac{\langle 1,0,1 \rangle \cdot \langle 1,1,0 \rangle}{2} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \boxed{\theta_A = \frac{\pi}{3}}$$

$\to$ The angle at B:

$$\cos\theta = \frac{\langle 0,1,-1 \rangle \cdot \langle -1,0,-1 \rangle}{2} \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \boxed{\theta_B = \frac{\pi}{3}}$$

$\to$ The angle at C:

$$\theta = \pi - \frac{\pi}{3} - \frac{\pi}{3} \Rightarrow \theta = \frac{3\pi}{3} - \frac{2\pi}{3} \Rightarrow \boxed{\theta_C = \frac{\pi}{3}}$$

$\to$ This is an equilateral triangle.

6. (12 points) Find the directional derivative of

$$f(x, y, z) = x^3 + y^3 + z^3 + xyz \quad ,$$

at the point $(1, 1, 1)$ in a direction pointing to the point $(-1, -1, -1)$.

---

ans. $-4\sqrt{3}$

$\rightarrow \nabla f = \langle 3x^2 + yz,\ 3y^2 + xz,\ 3z^2 + xy \rangle$

$\rightarrow Q - P = \langle -2, -2, -2 \rangle$

$\rightarrow |Q - P| = 2\sqrt{3}$

$\rightarrow u = \langle -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \rangle$

$\rightarrow \nabla f(1,1,1) = \langle 4, 4, 4 \rangle$

$\rightarrow \langle 4, 4, 4 \rangle \cdot \langle -\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \rangle = -\frac{4}{\sqrt{3}} - \frac{4}{\sqrt{3}} - \frac{4}{\sqrt{3}} = -\frac{12}{\sqrt{3}} = \frac{-12\sqrt{3}}{3} = \boxed{-4\sqrt{3}}$

7. (12 points) Using the Chain Rule (no credit for other methods), find

$$\frac{\partial g}{\partial u}$$

at $(u, v) = (0, 1)$, where

$$g(x, y) = 3x^2 - 3y^2 \quad ,$$

and

$$x = e^u \cos v \quad , \quad y = e^u \sin v \quad .$$

---

ans. $6\cos^2(1) - 6\sin^2(1)$

$\to g_u = (g_x)(x_u) + (g_y)(y_u)$

$\to g_x = 6x, \quad g_y = -6y$

$\to g_x = 6e^u \cos v, \quad g_y = -6e^u \sin v$

$\to x_u = e^u \cos v, \quad y_u = e^u \sin v$

$\to g_u = (6e^u \cos v)(e^u \cos v) - (6e^u \sin v)(e^u \sin v)$

$\to g_u(0,1) = (6\cos 1)(\cos 1) - (6\sin 1)(\sin 1)$

$\to \boxed{g_u(0,1) = 6\cos^2(1) - 6\sin^2(1)}$

8. (12 points) Without using Maple (or any other software), compute the vector-field surface integral $\int_S \mathbf{F}.d\mathbf{S}$ if

$$\mathbf{F} = \langle\, 3x + cos(y^3 + yz)\,,\, -2y + e^{x+z^2}\,,\, 5z + \sin(xy^3 + e^x)\,\rangle \qquad ,$$

and S is the closed surface in 3D space bounding the region

$$\{(x, y, z) : x^2 + y^2 + z^2 < 4 \quad and \quad x > 0 \quad and \quad y < 0 \quad and \quad z > 0\} \quad .$$

---

ans. 8π

→ $div(F) = 3 - 2 + 5$

→ $\iiint_D 6\, dV$

→ Since we have a constant:

→ $6\left(\frac{4}{3}\pi r^3\right) = \frac{24}{3}\pi r^3 = 8\pi r^3 \quad (r = 2)$

→ $8\pi\left(2^3\right) = 8\pi \cdot 8 = 64\pi$

→ Divide by 8:

→ $\boxed{8\pi}$

9. (12 points) Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if

$$\mathbf{F} = \langle 3z\,,\, 2x\,,\, y+z \rangle \quad ,$$

and S is the oriented surface

$$z = 2x + 3y \quad , \quad 0 < x < 1, \quad 0 < y < 1 \quad ,$$

with **upward pointing** normal.

---

ans. -15

$\to g(x,y) = 2x + 3y$

$\to g_x = 2 \,,\quad g_y = 3$

$\to \int_0^1\int_0^1 (-6z - 6x + y + z)\,dx\,dy$

$\to \int_0^1\int_0^1 (-6(2x+3y) - 6x + y + (2x+3y))\,dx\,dy$

$\to \int_0^1\int_0^1 (-12x - 18y - 6x + y + 2x + 3y)\,dx\,dy$

$\to \int_0^1\int_0^1 (-16x - 14y)\,dx\,dy$

$\to \int_0^1 (-16x - 14y)\,dx = -14y - 8$

$\to \int_0^1 (-14y - 8)\,dy = \boxed{-15}$

10. (12 points) Without using Maple or software, find the critical point(s) of

$$f(x,y) = 4x - y^2 - \ln(2x+y) \quad ,$$

and decide for each whether it is a local maximum, local minimum, or saddle point. Explain.

---

ans. $\left(\frac{3}{4}, -1\right)$ is a saddle point

$\rightarrow f_x = 4 - \frac{2}{2x+y}$

$\rightarrow f_y = -2y - \frac{1}{2x+y}$

$\rightarrow f_x = 0 \Rightarrow 4 - \frac{2}{2x+y} = 0 \Rightarrow \frac{4}{1} = \frac{2}{2x+y} \Rightarrow 4(2x+y) = 2 \Rightarrow 8x + 4y = 2$

$\rightarrow f_y = 0 \Rightarrow -2y - \frac{1}{2x+y} = 0 \Rightarrow \frac{-2y}{1} = \frac{1}{2x+y} \Rightarrow -2y(2x+y) = 1 \Rightarrow -4xy - 2y^2 = 1$

$\rightarrow$ Critical Point: $\left(\frac{3}{4}, -1\right)$

$\rightarrow f_{xx} = \frac{4}{(2x+y)^2}$ @ $\left(\frac{3}{4}, -1\right) = 16$

$\rightarrow f_{yy} = -2 + \frac{1}{(2x+y)^2}$ @ $\left(\frac{3}{4}, -1\right) = 2$

$\rightarrow f_{xy} = \frac{2}{(2x+y)^2}$ @ $\left(\frac{3}{4}, -1\right) = 8$

$\rightarrow D = f_{xx} f_{yy} - (f_{xy})^2$

$\rightarrow D = (16)(2) - (64) \Rightarrow D = -32 < 0$

$\rightarrow$ Since $D < 0$, we can say that our only critical point $\left(\frac{3}{4}, -1\right)$ is a saddle point.

11. (12 points) Without using Maple or software, using a **Linearization** around the point $(1,1,2)$, approximate $f(1.001, 0.999, 2.001)$ if

$$f(x,y,z) \; = \; \sqrt{2x^2+3y^2+z^2} \quad .$$

---

ans. $\frac{9001}{3000}$

$\rightarrow f_x = \frac{2x}{\sqrt{2x^2+3y^2+z^2}}$, $f_y = \frac{3y}{\sqrt{2x^2+3y^2+z^2}}$, $f_z = \frac{z}{\sqrt{2x^2+3y^2+z^2}}$

$\rightarrow f_x(1,1,2) = \frac{2}{\sqrt{2+3+4}} = \frac{2}{3}$

$\rightarrow f_y(1,1,2) = \frac{3}{3} = 1$

$\rightarrow f_z(1,1,2) = \frac{2}{3}$

$\rightarrow f(1,1,1) = 3$

$\rightarrow L(1.001, 0.999, 2.001) = 3 + \frac{2}{3}(1.001-1) + 1(0.999-1) + \frac{2}{3}(2.001-2)$

$\rightarrow \boxed{L(1.001, 0.999, 2.001) = \frac{9001}{3000}}$

12. (12 points) Without using Maple (or any other software) and by using polar coordinates (no credit for doing it directly) find

$$\int_0^{\frac{\sqrt{2}}{2}} \int_0^x x\,dy\,dx + \int_{\frac{\sqrt{2}}{2}}^1 \int_0^{\sqrt{1-x^2}} x\,dy\,dx \quad .$$

Explain!

---

ans. $\frac{1}{3\sqrt{2}}$

$\to 0 \le r \le \frac{\sqrt{2}}{2}\sec\theta$

$\to 0 \le \theta \le \pi/4$

$\to \int_0^{\pi/4}\int_0^{\frac{\sqrt{2}}{2}\sec\theta} r\cos\theta\, r\,dr\,d\theta$

$\to \frac{\sqrt{2}}{2}\sec\theta \le r \le 1$

$\to 0 \le \theta \le \frac{\pi}{4}$

$\to \int_0^{\pi/4}\int_{\frac{\sqrt{2}}{2}\sec\theta}^{1} r\cos\theta\, r\,dr\,d\theta$

$\to \int_0^{\pi/4}\int_0^{\frac{\sqrt{2}}{2}\sec\theta} r\cos\theta\, r\,dr\,d\theta + \int_0^{\pi/4}\int_{\frac{\sqrt{2}}{2}\sec\theta}^{1} r\cos\theta\, r\,dr\,d\theta = \boxed{\frac{1}{3\sqrt{2}}}$

13. (12 points) Convert the triple iterated integral

$$\int_0^{\cancel{4}\,2}\int_0^{\sqrt{4-z^2}}\int_{-\sqrt{4-z^2-y^2}}^{0} x^2\, y\, z\, dx\, dy\, dz$$

to **spherical coordinates**. **Do not evaluate.**

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ans. $\int_0^2\int_0^{\frac{\pi}{2}}\int_{\frac{\pi}{2}}^{\pi} \rho^6 \sin^4\phi \cos^2\theta \cos\phi \sin\theta \, d\theta\, d\phi\, d\rho$

→ Graphed on Maple

$$\rightarrow \int_0^2\int_0^{\frac{\pi}{2}}\int_{\frac{\pi}{2}}^{\pi} (\rho \sin\phi \cos\theta)^2 \rho \sin\phi \sin\theta\, \rho \cos\phi\, \rho^2 \sin\phi \, d\theta\, d\phi\, d\rho$$

$$\rightarrow \boxed{\int_0^2\int_0^{\frac{\pi}{2}}\int_{\frac{\pi}{2}}^{\pi} \rho^6 \sin^4\phi \cos^2\theta \cos\phi \sin\theta \, d\theta\, d\phi\, d\rho}$$

14. (12 points) Find the curvature of the curve

$$\mathbf{r}(t) = \langle 5, 3\sin t, 3\cos t \rangle$$

at the point where $t = \frac{\pi}{3}$.

---

ans. $\frac{1}{3}$

$\rightarrow r(t) = \langle 5, 3\sin t, 3\cos t \rangle$

$\rightarrow r'(t) = \langle 0, 3\cos t, -3\sin t \rangle$

$\rightarrow r''(t) = \langle 0, -3\sin t, -3\cos t \rangle$

$\rightarrow r'\left(\frac{\pi}{3}\right) = \langle 0, \frac{3}{2}, \frac{-3\sqrt{3}}{2} \rangle$

$\rightarrow r''\left(\frac{\pi}{3}\right) = \langle 0, -\frac{3\sqrt{3}}{2}, \frac{-3}{2} \rangle$

$\rightarrow r'\left(\frac{\pi}{3}\right) \times r''\left(\frac{\pi}{3}\right) = \langle 0, \frac{3}{2}, -\frac{3\sqrt{3}}{2} \rangle \times \langle 0, \frac{-3\sqrt{3}}{2}, \frac{-3}{2} \rangle = \langle -9, 0, 0 \rangle$

$\rightarrow |\langle -9, 0, 0 \rangle| = 9$

$\rightarrow \left|r'\left(\frac{\pi}{3}\right)\right|^3 = \left(\sqrt{0 + \frac{9}{4} + \frac{27}{4}}\right)^3 = \left(\sqrt{\frac{36}{4}}\right)^3 = 3^3 = 27$

$\rightarrow K\left(\frac{\pi}{3}\right) = \frac{9}{27}$

$\rightarrow \boxed{K\left(\frac{\pi}{3}\right) = \frac{1}{3}}$

15. (12 points) Set-up an iterated double integral, in type I format, but do not compute, for the surface area of the surface given parametrically by

$$\mathbf{r}(u,v) = \langle u^2 , uv , v^2 \rangle \quad , \quad 0 < u < v < 1 \quad .$$

---

ans. $\int_0^1 \int_0^v 2v\sqrt{v^2+4u^2}\, du\, dv$

$\rightarrow \iint_D \| N(u,v) \| \, du\, dv$

$\rightarrow \begin{vmatrix} & & \\ 2u & v & 0 \\ 0 & u & 2v \end{vmatrix}$

$\rightarrow 2v^2 i - 4uv j = \langle 2v^2, -4uv, 0 \rangle$

$\rightarrow | \langle 2v^2, -4uv, 0 \rangle | = \sqrt{4v^4 + 16u^2v^2} = \sqrt{4v^2(v^2+4u^2)} = 2v\sqrt{v^2+4u^2}$

$\rightarrow \boxed{\int_0^1 \int_0^v 2v\sqrt{v^2+4u^2}\, du\, dv}$

16. (12 points) Let

$$f(x, y, z) = xy^2z^3 \quad ,$$

and let

$$g(x, y, z) = x + y^2 + z^3 \quad .$$

compute the dot-product

$$grad(f) \,.\, grad(g) \qquad .$$

at the point $(1, 1, 1)$.

---

ans. 14

$\rightarrow \nabla f = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$

$\rightarrow \nabla g = \langle 1, 2y, 3z^2 \rangle$

$\rightarrow \nabla f(1,1,1) = \langle 1, 2, 3 \rangle$

$\rightarrow \nabla g(1,1,1) = \langle 1, 2, 3 \rangle$

$\rightarrow \langle 1, 2, 3 \rangle \cdot \langle 1, 2, 3 \rangle = \boxed{14}$

17. (8 points) Decide whether the following limit exists. If it does ,find it. If it does not, explain why it does not exist.

$$\lim_{(x,y,z,w)\to(0,0,0,0)} \frac{(x+y)^2-(z+w)^2}{x+y-z-w} \quad .$$

---

ans. 0

$\rightarrow \lim_{(x,y,z,w)\to(0,0,0,0)} \frac{(x+y)^2-(z+w)^2}{x+y-z-w} = \frac{0}{0}$

$\rightarrow$ The numerator is a difference of squares:

$\rightarrow \lim_{(x,y,z,w)\to(0,0,0,0)} \frac{(x+y+z+w)\cancel{(x+y-z-w)}}{\cancel{(x+y-z-w)}}$

$\rightarrow \lim_{(x,y,z,w)\to(0,0,0,0)} (x+y+z+w) = \boxed{0}$