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SCC 1

SCC 2

1. vector field not conservative (curl F $\neq$ <0,0>)

2. $\int_{\frac{1}{2}}^{1} \int_{\frac{1}{4}}^{1-y^2} f(x,y)\,dx\,dy$

3. $z = -\frac{1}{2}x - \frac{5}{6}y + \frac{7}{18}\pi$

4. $3\hat{i} - 8\hat{j} + 4\hat{k}$

5. Angle A: $60°$; $\frac{\pi}{3}$

Angle B: $60°$; $\frac{\pi}{3}$

Angle C: $60°$; $\frac{\pi}{3}$

6. $\frac{10}{\sqrt{3}}$

7. $6\cos^2(1) - 6(\sin^2(1))$

8.

9. 15

10. ~~en~~ only critical point : $(\frac{3}{4}, -1)$ = local min ($f_{xx} > 0$ and $D > 0$

11. $\frac{9003}{3000} = 3.001$

12. $\frac{5\sqrt{2}}{16}$

13. $\int_0^{\pi} \int_0^{\frac{\pi}{2}} \int_{-2}^{0} p^6 (\sin^4\phi \cos\phi)(\cos^2\theta \sin\theta)\, dp\, d\theta\, d\phi$

14. $\frac{1}{3}$

15. $\int_0^1 \int_0^v \sqrt{4v^4 + 16u^2v^2 + 4u^4}\, du\, dv$

16. 13

17. 0

Problem #7

Problem #1.

$$\int_c \left(\cos\left(e^{\sin x}\right) + 5y \right) dx + \left(\sin\left(e^{\cos y}\right) + 11x \right) dy$$

Parametrize the paths:

Line #1:

$x = (1-t)(1) + t(-1) = 1 - t - t = 1 - 2t$

$y = (1-t)(0) + t(0) = 0$

Line #2:

$x = (1-t)(-1) + t(-1) = -1 + t - t = -1$

$y = (1-t)(0) + t(1) = t$

Line #3:

$x = (1-t)(-1) + 0 = -1 + t$

$y = (1-t) + 2(t) = 1 - t + 2t = 1 + t$

Line #4:

$x = (1-t)(0) + t = t$

$y = (1-t)(2) + t = 2 - 2t + t = 2 - t$

Line #5:

$x = (1-t) + t = 1$

$y = (1-t) + 0 = 1 - t$

Line 1:

$x = 1-2t$

$y = 0$

$$\int_0^1 \left(\cos\left(e^{\sin(1-2t)}\right)(-2) + 0\, dt$$

Line 2:

$x = -1$

$y = t$

$$\int_0^1 \left(\cos\left(e^{\sin(-1)}\right)(0) + \sin\left(e^{\cos(t)}\right) + (-11)\, dt$$

Line 3:

$x = -1+t$

$y = 1+t$

$$\int_0^1 \left(\cos\left(e^{\sin(-1+t)}\right)(1) + \sin\left(e^{\cos(1+t)}\right) + 11(-1+t)\right)$$

Line 4.

$x = t$

$y = 2-t$

Set up integrals and solve, then add them all together to obtain.

Line 5:

$x = 1$

$y = 1-t$

curl $\vec{F}$

$$\begin{vmatrix} \frac{d}{dx} & \frac{d}{dy} \\ \cos(e^{\sin x}) + 5y & \sin(e^{\cos y}) + 11x \end{vmatrix}$$

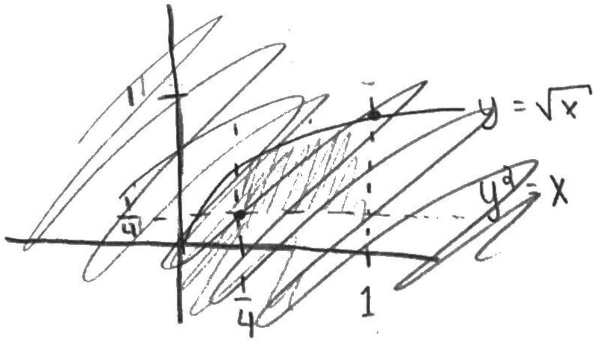
↳ curl $\vec{F} \neq 0$

vector field is not conservative so it is not possible.

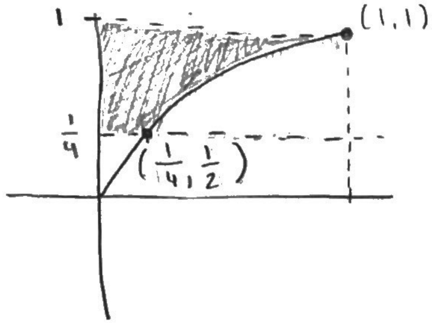
Problem #2.

$$\int_{\frac{1}{4}}^{1} \int_{0}^{\sqrt{x}} f(x,y)\,dy\,dx$$

$$\cancel{\boxed{\int_{0}^{1} \int_{\frac{1}{4}}^{y^2} f(x,y)\,dx\,dy}}$$



$y^2 = x$



$$\boxed{\int_{\frac{1}{2}}^{1} \int_{\frac{1}{4}}^{1-y^2} f(x,y)\,dx\,dy}$$

Problem #3.

eq of tangent line = $a(x-x_0) + b(y-y_0) + c(z-z_0) = d$

$(x_0, y_0, z_0) = \left(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}\right)$

$$\nabla F = \langle -2\sin(x+y) - 4\sin(x+z),\ -2\sin(x+y) - 8\sin(y+z),\ -4\sin(x+z) - 8\sin(y+z) \rangle$$

$$\nabla F\left(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}\right) = \left\langle -2\sin\left(\frac{\pi}{3}\right) - 4\sin\left(\frac{\pi}{3}\right),\ -2\sin\left(\frac{\pi}{3}\right) - 8\sin\left(\frac{\pi}{3}\right),\ -4\sin\left(\frac{\pi}{3}\right) - 8\sin\left(\frac{\pi}{3}\right) \right\rangle$$

$$= \left\langle -2\frac{\sqrt{3}}{2} - 4\frac{\sqrt{3}}{2},\ -2\frac{\sqrt{3}}{2} - 8\frac{\sqrt{3}}{2},\ -4\frac{\sqrt{3}}{2},\ -8\sin\frac{\sqrt{3}}{2} \right\rangle$$

$$= \left\langle -\frac{6\sqrt{3}}{2},\ -\frac{10\sqrt{3}}{2},\ -\frac{12\sqrt{3}}{2} \right\rangle = \langle -3\sqrt{3}, -5\sqrt{3}, -6\sqrt{3} \rangle$$

Tangent Line:

$$-3\sqrt{3}\left(x-\frac{\pi}{6}\right)-5\sqrt{3}\left(y-\frac{\pi}{6}\right)-6\sqrt{3}\left(z-\frac{\pi}{6}\right)=0$$

$$-3\sqrt{3}\,x+\frac{3\sqrt{3}\,\pi}{6}-5\sqrt{3}\,y+\frac{5\sqrt{3}\,\pi}{6}-6\sqrt{3}\,z+\frac{6\sqrt{3}\,\pi}{6}=0$$

$$3\sqrt{3}\,x+5\sqrt{3}\,y+6\sqrt{3}\,z=\frac{14\pi}{6}\sqrt{3}$$

$$3x+5y+6z=\frac{14\pi}{6}$$

$$6z=-3x-5y+\frac{14\pi}{6}$$

$$\boxed{z=-\frac{1}{2}x-\frac{5}{6}y+\frac{14\pi}{36}}$$

Problem #4.

$a \times (-b) + a \times c + b \times 2a + b \times 3c + c \times 2a + c \times -6$

$(-i - j + k) + 2i + j + 2k - (2i + 2j - k) + 3i - 3j + 3k$

$-(4i + 2j + 4k) - (-i + j - k)$

$$\left.\begin{aligned} &i(-1+2+2+3-4+1) = 3 \\ &j(-1+1-2-3-2-1) = -8 \\ &k(1+2+1+3-4+1) = 4 \end{aligned}\right\} \quad \boxed{3\hat{i} - 8\hat{j} + 4\hat{k}}$$

Problem #6.

Problem #5.

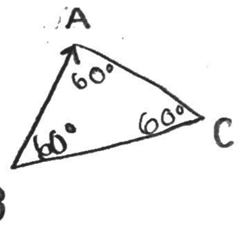
$A = (0,0,0)$

$B = (1,0,1)$

$C = (1,1,0)$

$$\cos\theta = \frac{a \cdot b}{\|a\| \|b\|}$$

$\cos\theta$



$$\vec{AB} = (0-1, 0-0, 0-1) = \langle -1, 0, -1 \rangle$$

$$\vec{BC} = \langle 1-1, 1-0, 0-1 \rangle = \langle 0, 1, -1 \rangle$$

$$\cos\theta = \frac{AB \cdot BC}{\|AB\| \|BC\|}$$

$$\cos\theta = \frac{(-1)(0) + (0)(1) + (-1)(-1)}{\sqrt{2} \times \sqrt{\sqrt{2}}}$$

$$\cos\theta = \frac{1}{2}$$

$$\cos^{-1}\left(\tfrac{1}{2}\right) = 60^\circ$$

$$\vec{CB} = (1-1, 0-1, 1-0) = \langle 0, -1, 1 \rangle$$

$$\vec{CA} = \langle 0-1, 0-1, 0-0 \rangle = \langle -1, -1, 0 \rangle$$

$$\cos\theta = \frac{CB \cdot CA}{\|CB\| \|CA\|} = \frac{(0)(-1) + (-1)(-1) + (1)(0)}{\sqrt{2} \times \sqrt{2}} = \frac{1}{2}$$

$$\cos^{-1}\left(\tfrac{1}{2}\right) = 60^\circ$$

$$\boxed{\begin{array}{l} \text{Angle A: } 60^\circ;\ \frac{\pi}{3} \\ \text{Angle B: } 60^\circ;\ \frac{\pi}{3} \\ \text{Angle C: } 60^\circ;\ \frac{\pi}{3} \end{array}}$$

$$\text{radians} = 60^\circ \times \frac{\pi}{180} = \frac{\pi}{3}$$

Problem #6.

F $\nabla f = \langle f_x, f_y, f_z \rangle$

z $f_x = 3x^2 + yz$

y $f_y = 3y^2 + xz$

$f_z = 3z^2 + xy$

$\vec{v} = \langle -1-1, -1-1, -1-1 \rangle$

$= \langle -2, -2, -2 \rangle$

$\|\vec{v}\| = \sqrt{2^2 + 2^2 + 2^2}$

$= \sqrt{12} = 2\sqrt{3}$

$\hookrightarrow u = \left\langle \frac{1}{2\sqrt{3}}, \frac{2}{2\sqrt{3}}, \frac{2}{2\sqrt{3}} \right\rangle$

$= \left\langle \frac{1}{2\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$

$\nabla f(1,1,1) = \langle 3+1, 3+1, 3+1 \rangle = \langle 4,4,4 \rangle$

$\nabla f(1,1,1) \cdot u = \langle 4,4,4 \rangle \cdot \left\langle \frac{1}{2\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle$

$= \frac{4}{2\sqrt{3}} + \frac{4}{\sqrt{3}} + \frac{4}{\sqrt{3}}$

$= \frac{4}{2\sqrt{3}} + \frac{8}{2\sqrt{3}} + \frac{8}{2\sqrt{3}} = \frac{20}{2\sqrt{3}} = \boxed{\frac{10}{\sqrt{3}}}$

$\left(\frac{10}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{10\sqrt{3}}{3} \right)$

Problem #7.

$\frac{dg}{du} = \frac{dg}{dx}\frac{dx}{du} + \frac{dg}{dy}\frac{dy}{du}$

$\frac{dg}{du} = (6x)(e^u \cos v) + (-6y)(e^u \sin v)$

$= 6(e^u \cos v)(e^u \cos v) - 6(e^u \sin v)(e^u \sin v)$

@ $(u,v) = (0,1)$

$= 6(e^0 \cos(1))(e^0 \cos(1)) - 6(e^0 \sin(1))(e^0 \sin(1)) =$

$= \boxed{6\cos^2(1) - 6(\sin^2(1))}$

Problem #8:

$\int_S F \cdot dS$

$F = \langle 3x + \cos(y^3 + yz), -2y + e^{x+z^2}, 5z + \sin(xy^3 + e^x) \rangle$

$\boxed{0 < p < 2}$

$p\sin\phi \sin\theta < 0$

$p\cos\phi > 0$

$\cancel{0 < \phi < \pi}$

$p\sin\phi\cos\theta < 0$

$0 < \sin\phi\cos\theta < \frac{\pi}{2}$

$\rightarrow 0 < \phi < \frac{\pi}{2}$

$\rightarrow 0 < \theta < \frac{\pi}{2}$

$0 \leq z \leq \sqrt{4 - x^2 - y^2}$

$-\sqrt{4 - x^2} \leq y \leq 0$

$0 \leq x \leq 2$

$$\int_0^2 \int_{-\sqrt{4-x^2}}^0 -9x + 3\cos(y^3 + yz) - 4y^2 + 2ye^{x+z^2} + 5z + \sin(xy^3 + e^x)\, dy\, dx$$

Problem # 9.

$F = \langle 3z, 2x, y+z \rangle$

$z = 2x + 3y \quad 0 < x < 1 \quad 0 < y < 1$

$$\iint_S F \cdot dS = \iint_D \left(-P\frac{dg}{dx} - Q\frac{dg}{dy} + R\right) dA$$

$$= \int_0^1\int_0^1 (3z)(2) - 2x(3) + (y+z)\, dx\, dy$$

$\Rightarrow$ inner int:

$$\int_0^1 6(2x+3y) - 6x + y + 2x + 3y \; dx$$

$$= \int_0^1 12x + 18y - 6x + y + 2x + 3y \; dx$$

$$= \int_0^1 [8x + 22y]\, dx = [4x^2 + 22yx]\Big|_0^1$$

$$= [4 + 22y]$$

outer int:

$$\int_0^1 (4 + 22y)\, dy = [4y + 11y^2]\Big|_0^1 = 4 + 11 = \boxed{15}$$

Problem #10.

$f(x,y) = 4x - y^2 - \ln(2x+y)$

$f_x = 4 - \frac{2}{2x+y} = 0 \Rightarrow 4 = \frac{2}{(2x+y)} \Rightarrow 8x + 4y = 2$

$4x + 2y = 1$

$f_y = -2y - \frac{1}{2x+y} = 0$

$2y = 1 - 4x$

$y = \frac{1}{2} - 2x$

$-2y = \frac{1}{2x+y}$

$-2\left(\frac{1}{2} - 2x\right) = \frac{1}{\cancel{2x} + \frac{1}{2} - \cancel{2x}}$

$-1 + 4x = \frac{1}{\frac{1}{2}}$

$\frac{-1}{2} + 2x = 1$

$2x = \frac{3}{2}$

$4x = 3$

$x = \frac{3}{4} \rightarrow y = \frac{1}{2} - 2\left(\frac{3}{4}\right)$

$= \frac{1}{2} - \frac{6}{4} = \frac{1}{2} - \frac{3}{2} = \frac{-2}{2} = -1$

$\left(\frac{3}{4}, -1\right)$

critical point: $(\frac{3}{4}, -1)$

$f_{xx} = \frac{d}{dx}\left(-2(2x+y)^{-1}\right) = 2(2x+y)^{-2}(2) = \frac{4}{(2x+y)^2}$

$f_{yy} = \frac{d}{dy}\left(-2y - (2x+y)^{-1}\right) = \left(-2 + (2x+y)^{-2}\right) = -2 + \frac{1}{(2x+y)^2}$

$f_{xy} = \frac{d}{dy}\left(4 - \frac{2}{2x+y}\right) = \frac{d}{dy}\left(-2(2x+y)^{-1}\right)$

$= +2(2x+y)^{-2}(1) = \frac{2}{(2x+y)^2}$

$D(\frac{3}{4}, -1) = \left(\frac{4}{(2x+y)^2}\right)\left(\frac{1}{(2x+y)^2}\right) - \left(\frac{2}{(2x+y)^2}\right)$ (16) (4)

$= (16)(4) - (8)^2 = 64 - 64 = 0$

inconclusive.

$D(\frac{3}{4}, 1) = \left(\frac{4}{(\frac{6}{4}-1)^2}\right)\left(-2 + \frac{1}{(\frac{6}{4}-1)^2}\right) - \left(\frac{2}{(\frac{6}{4}-1)^2}\right)$

$= \left(\frac{4}{1/4}\right)(-2+4) - \left(\frac{2}{1/4}\right) = 16(2) - 8) = 32 - 8 = 24 > 0$

$f_{xx}(\frac{3}{4}, -1) = \frac{4}{(2x+y)^2} = \frac{4}{1/4} = 16 > 0$

Since $f_{xx} > 0$ and $D > 0$, $f(\frac{3}{4}, -1)$ is a local minimum

Problem #11.

$$f(x,y,z) = \sqrt{2x^2+3y^2+z^2} = (2x^2+3y^2+z^2)^{1/2}$$

$$f_x = \frac{1}{2}(2x^2+3y^2+z^2)^{-1/2}(4x)$$

$$f(1,1,2) = \sqrt{2+3+2^2}$$
$$= 3$$

$$f_y = \frac{1}{2}(2x^2+3y^2+z^2)^{-1/2}(6y)$$

$$f_z = \frac{1}{2}(2x^2+3y^2+z^2)^{-1/2}(2z)$$

$$f_x(1,1,2) = \frac{2x}{\sqrt{2x^2+3y^2+z^2}} = \frac{2(1)}{\sqrt{2+3+4}} = \frac{2}{\sqrt{9}} = \frac{2}{3}$$

$$f_y(1,1,2) = \frac{3y}{\sqrt{2x^2+3y^2+z^2}} = \frac{3}{3} = 1$$

$$f_z(1,1,2) = \frac{2z}{\sqrt{2x^2+3y^2+z^2}} = \frac{4}{3}$$

all continuous at $(1,1,2)$ so the linearization is...

$$L(x,y,z) = 3 + \frac{2}{3}(x-1) + (y-1) + \frac{4}{3}(z-2)$$

$$L(1.001, 0.999, 2.001) = 3 + \frac{2}{3}(1.001-1) + (0.999-1) + \frac{4}{3}(2.001-2)$$

$$= 3 + \frac{2}{3000} - \frac{1}{1000} + \frac{4}{3000}$$

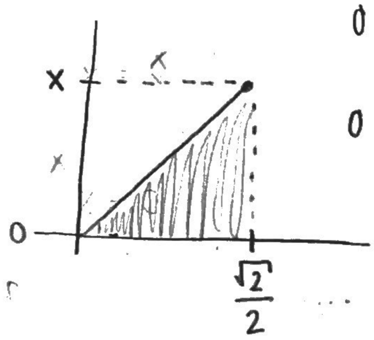
$$= \frac{9000+2-3+4}{3000} = \boxed{3.001 = \frac{9003}{3000}}$$

Problem 12.

$$\int_0^{\frac{\sqrt{2}}{2}} \int_0^x x\,dy\,dx + \int_{\frac{\sqrt{2}}{2}}^1 \int_0^{\sqrt{1-x^2}} x\,dy\,dx$$

$y = \sqrt{1-x^2}$

Sketch:



$0 \le \theta \le \frac{\pi}{4}$

$0 \le r \le \sec\theta$

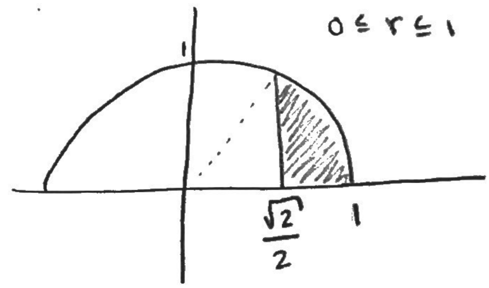
$\frac{\sqrt{2}}{2} = r\cos\theta$

$r = \frac{\sqrt{2}}{2}\sec\theta$

Sketch:

$0 \le \theta \le \frac{\pi}{4}$

$0 \le r \le 1$



$$\int_0^{\frac{\pi}{4}} \int_0^1 r\cos\theta\,dr\,d\theta$$

$$\int_0^{\frac{\pi}{4}} \int_0^{\frac{\sqrt{2}}{2}\sec\theta} r\cos\theta\,dr\,d\theta + \int_0^{\frac{\pi}{4}} \int_0^1 r\cos\theta\,dr\,d\theta$$

$$\int_0^{\frac{\pi}{4}} \left(\frac{r^2}{2}\cos\theta\right)\Big|_0^{\frac{\sqrt{2}}{2}} d\theta + \int_0^{\frac{\pi}{4}} \left(\frac{r^2}{2}\cos\theta\right)\Big|_0^1 d\theta$$

$$\frac{1}{4}\int_0^{\frac{\pi}{4}} \cos\theta\,d\theta + \frac{1}{2}\int_0^{\frac{\pi}{4}} \left(\frac{r^2}{2}\cos\theta\right)\Big|_0^1 d\theta$$

$$\left(\frac{1}{4}+\frac{1}{2}\right)(\sin\theta)\Big|_0^{\frac{\pi}{4}} = \frac{5}{8}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{5\sqrt{2}}{16}}$$

Problem #13.

$$\int_0^4 \int_0^{\sqrt{4-z^2}} \underbrace{\int_{-\sqrt{4-z^2-y^2}}^{0}}_{1/2 \text{ sphere!}} x^2yz \, dx\,dy\,dz$$

$$\int_0^{\pi} \int_0^{\pi} \int_{-2}^{0} \rho^2 \sin^2\phi \cos^2\theta \, \rho\sin\phi \, \sin\theta \, \rho\cos\phi \, \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$$

$$\boxed{\int_0^{\pi} \int_0^{\frac{\pi}{2}} \int_{-2}^{0} \rho^6 (\sin^4\phi \cos\phi)(\cos^2\theta \sin\theta) \, d\rho \, d\phi \, d\theta}$$

Problem #14.

$r(t) = \langle 5, 3\sin t, 3\cos t \rangle$

$r'(t) = \langle 0, 3\cos t, -3\sin t \rangle$

$r''(t) = \langle 0, -3\sin t, -3\cos t \rangle$

$$r'(t) \times r''(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3\cos t & -3\sin t \\ 0 & -3\sin t & -3\cos t \end{vmatrix} = (-9\cos^2(t) - 9\sin^2(t))\,\hat{i} - 0\,\hat{j} + 0\,\hat{k}$$

$$= -9(\cos^2 t + \sin^2 t) = -9\,\hat{i}$$

$$r'(t) = \sqrt{9\cos^2 t + 9\sin^2 t} = 3$$

$$K(t) = \frac{|-9|}{3^3} = \frac{9}{27} = \boxed{\frac{1}{3}}$$

Problem #15.

$r(u,v) = \langle u^2, uv, v^2 \rangle$

0

$A = \iint_D \|r_u \times r_v\| \, dA$

$N = \frac{dr}{du} \times \frac{dr}{dv}$

$r_u = \langle 2u, v, 0 \rangle$

$r_v = \langle 0, u, 2v \rangle$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2u & v & 0 \\ 0 & u & 2v \end{vmatrix} = (2v^2 - 0)\hat{i} \mp (4uv - 0)\hat{j} + (2u^2 - 0)\hat{k}$

$\langle 2v^2, -4uv, 2u^2 \rangle$

$\|r_u \times r_v\| = \sqrt{(2v^2)^2 + (-4uv)^2 + (2u^2)^2} = \underline{2\sqrt{v^4 + 4u^2v^2 + u^4}}$

$= \cancel{\sqrt{4v^4 + 16u^2v^2 + 4u^4}} = \cancel{2\sqrt{v^2 + u^2v^2 + u^4}}$

$\cancel{\iint 2\sqrt{v^4 + 4u^2v^2 + u^4}}$

$$\int_0^1 \int_0^v \sqrt{4v^4 + 16u^2v^2 + 4u^4} \, du\,dv$$

Problem #16.

$$\nabla(f) = \begin{bmatrix} y^2z^3 \\ 2xyz^3 \\ 3xy^2z^2 \end{bmatrix} \qquad \nabla(g) = \begin{bmatrix} 0 \\ 2y \\ 3z^2 \end{bmatrix}$$

$$\nabla(f) \cdot \nabla(g) = 0 + 4xy^2z^3 + 9xy^2z^4$$

$$@\ (1,1,1) = 4(1\times1\times1) + 9(1\times1\times1) = \boxed{13}$$

Problem #17.

$$\lim_{(x,y,z,w)\to(0,0,0,0)} \frac{(x+y)^2 - (z+w)^2}{x+y-z-w}$$

CANNOT plug in directly because it yields an indeterminant

in maple...

$$\lim\left(\frac{((x+y)^2 - (z+w)^2)}{(x+y-z-w)}, \{x=0, y=0, z=0, w=0\}\right);$$

Answer = 0