NAME: (print!) RUID: (print!) SSC: (circle) None / I / II / I and II

MATH 251 (22,23,24) [Fall 2020], Dr. Z. , Final Exam , Tue., Dec. 15, 2020

Email the completed test, renamed as finalFirstLast.pdf to DrZcalc3@gmail.com no later than 3:30pm, (or, in case of conflict, three and half hours after the start).

WRITE YOUR FINAL ANSWERS BELOW

1. -18

2. ½ <= y <=1, y^2 <= x <=1

3. -3sqrt2(x- (pi/3)) + -5sqrt3(y - (pi/3)) + -6sqrt3(z-(pi/3))

4. <-1,-6,6>

5. <A= pi/3

   <B= pi/3

   <C= pi/3

6. -4sqrt3

7.6cos^2(1)-6sin^2(1)

8. 8pi

9. -15

10. Saddle point-(¾,-1)

11. 3.0003

12. 1/3

13. int_(0,2) p^6 * int(0,pi/2) sin^4φ*cosφ dφ * int(pi,pi/2) cos^2**θ***sin**θ**d**θ**

14. 1/3

15. Int_(0,1) Int _(0,1) sqrt(4v^4 +16u^2*v^2 + 4u^2)du dv

16. 14

17. 0

Sign the following declaration:

I Hereby declare that all the work was done by myself. I was allowed to use Maple (unless specifically told not to), calculators, the book, and all the material in the web-page of this class but not other resources on the internet.

I only spent (at most) 3 hours on doing the exam. The last 30 minutes were spent in checking and double-checking the answers.

I also understand that I may be subject to a random short chat to verify that I actually did it all by myself.

Signed:

Possibly useful facts from school Geometry (that you are welcome to use) : (i) The area of a circle radius r is πr^2. (ii) The circumference of a circle radius r is $2\pi r$ (iii) The parametric equation of an ellipse with axes a b and parallel to the x and y axes respectively is $x = a \cos\theta$, $y = b \cos\theta$, $0 < \theta < 2\pi$. (iv) The area of an ellipse with axes a and b is πab (v) The volume and surface area of a sphere radius R are $\frac{4}{3}\pi R^3$ and $4\pi R^2$ respectively (vi) The volume of a cone is the area of the base times the height over 3. (vii) The volume of a pyramid is the area of the base times the height over 3. (viii) The area of a triangle is base times height over 2.

Formula that you may (or may not) need

If the surface S is given in explicit notation $z = g(x, y)$, above the region of the xy-plane ' D, then

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_D -P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R$$

dA .

2

1. (12 pts.) Without using Maple (or any software) Compute the vector-field line integral

$$\int_C (\cos(e^{\sin x}) + 5y)\, dx + (\sin(e^{\cos y}) + 11x)\, dy \,,$$

over the path consisiting of the five line segments (in that order)

$$(1, 0) \to (-1, 0) \to (-1, 1) \to (0, 2) \to (1, 1) \to (1, 0) \,.$$

Explain!

1)

$dx\,dy = 2D$

Use greens theorem $\iint Q_x - P_y \, dA$

$\iint 11 - 5 \, dA \rightarrow 6$

$\triangle \rightarrow \left(\frac{1}{2} \cdot 2\right) \cdot 1 = 1$

$\square \rightarrow 2 \cdot 1 = 2$ $\Rightarrow 1 + 2 = 3$

$3 \cdot 6 = 18$

$\updownarrow$

-18 (clock wise)

ans.

ans.

2. (12 points) Change the order of integration

$$\int_{\frac{1}{4}}^{1} f(x, y)\, dx\, dy .$$

ans.

$Z^{\sqrt{}}_{x}\, 0$

4

3. (12 points) Find the equation of the tangent plane at the point $(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6})$ to the surface

given implicitly by

$$2\cos(x + y) + 4\cos(x + z) + 8\cos(y + z) = 7\ .$$

Express you answer in explicit form, i.e in the format $z = ax + by + c$.

ans. $z =$

4. (16 points) Let a, b, c be three vectors such that

$$a \times b = i + j - k\ ,\ b \times c = i - j + k\ ,\ a \times c = 2i + j + 2k\ .$$

What is

$$(a + b + c) \times (2a - b + 3c)\ ?$$

ans.

5. (12 points) Find the three angles of the triangle ABC where $A = (0, 0, 0)$, $B = (1, 0, 1)$, $C = (1, 1, 0)$.

ans. The angle at A is: radians ;

The angle at B is: radians ;

The angle at C is: radians ;

6. (12 points) Find the directional derivative of

$$f(x, y, z) = x^3 + y^3 + z^3 + xyz ,$$

at the point (1, 1, 1) in a direction pointing to the point (−1, −1, −1) .

ans.

7. (12 points) Using the Chain Rule (no credit for other methods), find

$$\frac{\partial g}{\partial u}$$

at $(u, v) = (0, 1)$, where

$$g(x, y) = 3x^2 - 3y^2,$$

and

$$x = e^u \cos v\,,\; y = e^u \sin v\,.$$

ans.

8. (12 points) Without using Maple (or any other software), compute the vector-field surface integral R$_S$F.*d*S if

$$F = h\ 3x + cos(y^3 + yz),\ -2y + e^{x+z_2},\ 5z + \sin(xy^3 + e^x)i\ ,$$

and *S* is the closed surface in 3D space bounding the region

$$\{(x, y, z) : x^2 + y^2 + z^2 < 4 \text{ and } x > 0 \text{ and } y < 0 \text{ and } z > 0\}\ .\ \text{ans.}$$

9. (12 points) Compute the vector-field surface integral R R$_S$F.*d*S if

$$F = h\ 3z\ ,\ 2x\ ,\ y + z\ i\ ,$$

and S is the oriented surface

$$z = 2x + 3y\ ,\ 0 < x < 1,\ 0 < y < 1\ ,$$

with upward pointing normal.

ans.

10. (12 points) Without using Maple or software, find the critical point(s) of

$$f(x, y) = 4x - y^2 - \ln(2x + y) ,$$

and decide for each whether it is a local maximum, local minimum, or saddle point. Ex plain.

ans.

11. (12 points) Without using Maple or software, using a Linearization around the point

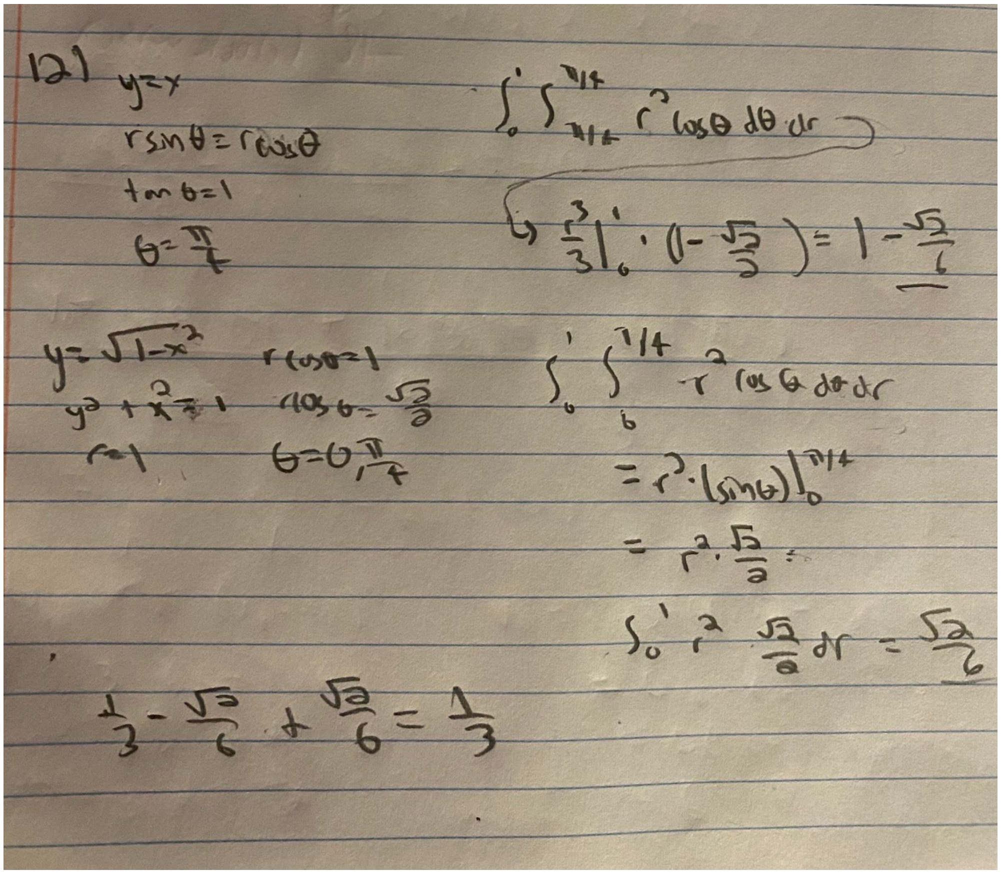
(1, 1, 2), approximate $f(1.001, 0.999, 2.001)$ if

$$f(x, y, z) = \sqrt{2x^2 + 3y^2 + z^2}.$$

ans.

12. (12 points) Without using Maple (or any other software) and by using polar coordi

nates (no credit for doing it directly) find



12) $y=x$

$r\sin\theta = r\cos\theta$

$\tan\theta = 1$

$\theta = \frac{\pi}{4}$

$\int_0^1 \int_{\pi/4}^{\pi/4} r^2 \cos\theta \, d\theta \, dr$

$\hookrightarrow \frac{r^3}{3}\Big|_0^1 \cdot \left(1 - \frac{\sqrt{2}}{2}\right) = 1 - \frac{\sqrt{2}}{6}$

$y = \sqrt{1-x^2}$ $\quad r\cos\theta = 1$

$y^2 + x^2 = 1$ $\quad \cos\theta = \frac{\sqrt{2}}{2}$

$r = 1$ $\quad \theta = 0, \frac{\pi}{4}$

$\int_0^1 \int_0^{\pi/4} r^2 \cos\theta \, d\theta \, dr$

$= r^2 \cdot (\sin\theta)\Big|_0^{\pi/4}$

$= r^2 \cdot \frac{\sqrt{2}}{2}$

$\int_0^1 r^2 \frac{\sqrt{2}}{2} \, dr = \frac{\sqrt{2}}{6}$

$\frac{1}{3} - \frac{\sqrt{2}}{6} + \frac{\sqrt{2}}{6} = \frac{1}{3}$

$x\,dy\,dx$. 0 Z x 0 $x\,dy\,dx$ + Z 1 $\sqrt{2}$ 2 Z $\sqrt{1-x^2}$ 0

ans.

Explain! Z $\sqrt{2}$ 2 2

13. (12 points) Convert the triple iterated integral

$$\int_0^4 \int_0^{\sqrt{4-z^2}} \int_{-\sqrt{4-z^2-y^2}}^{0} x^2 y z \, dx\, dy\, dz$$

to spherical coordinates. Do not evaluate. ans.

14. (12 points) Find the curvature of the curve r(*t*) = *h*5,

3 sin *t,* 3 cos*ti*

at the point where $t = \frac{\pi}{3}$.

ans.

15. (12 points) Set-up an iterated double integral, in type I format, but do not compute, for the surface area of the surface given parameterically by

$$r(u, v) = hu^2, uv\ , v^2 i\ , 0 < u < v < 1\ .$$

$$f(x, y, z) = xy^2z^3,$$

ans.

16. (12 points) Let

$$f(x, y, z) = xy^2z^3,$$

and let

$$g(x, y, z) = x + y^2 + z^3.$$

compute the dot-product

$$grad(f).\ grad(g)\ .$$

at the point (1, 1, 1).

ans.

17. (8 points) Decide whether the following limit exists. If it does ,find it. If it does not, explain why it does not exist.

$(x + y)^2 - (z + w)^2$

$$x + y - z - w\cdot$$

ans.

17)

$\lim_{x \to 0} \frac{x^2}{x} = x = 0$ $\lim_{y \to 0} \frac{y^2}{y} = y = 0$

$\lim_{z \to 0} \frac{-z^2}{-z} = z = 0$ $\lim \frac{-w^2}{w} = -w = 0$

Limit existes $\rightarrow$ $L = 0$

19

lim

$(x,y,z,w) \to (0,0,0,0)$