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SSC: (circle) None / I / II / I and II

MATH 251 (22,23,24) [Fall 2020], Dr. Z. , Final Exam , Tue., Dec. 15, 2020

Email the completed test, renamed as `finalFirstLast.pdf` to DrZcalc3@gmail.com no later than 3:30pm, (or, in case of conflict, three and half hours after the start).

WRITE YOUR FINAL ANSWERS BELOW

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1. -30
2. $\int_0^{1/2}\int_{1/4}^{1} f(x,y)\,dx\,dy + \int_{1/2}^{1}\int_{y^2}^{1} f(x,y)\,dx\,dy$
3. $z = -x/2 - 5y/6 + 7\pi/18$
4. $3\hat{i} - 6\hat{j} + 9\hat{k}$
5. $\pi/3$ for each angle
6. $-12/\sqrt{3}$
7. $6\cos(1) - 6\sin(1)$
8. 8π
9. 15
10. $(3/4, -1)$ is saddle point
11. $9001/3000$
12. $\sqrt{2}/6$
13. $\int_0^{\pi/2}\int_{\pi/2}^{\pi}\int_0^{2} (\rho\sin\phi\cos\theta)^2(\rho\sin\phi\sin\theta)(\rho\cos\phi)\,\rho^2\sin\phi\,d\rho\,d\theta\,d\phi$
14. $1/3$
15. $\int_0^1\int_0^v 2\sqrt{v^4+4u^2v^2+u^4}\,du\,dv$
16. 14
17. 0

Sign the following declaration:

I Ashwin Haridas Hereby declare that all the work was done by myself. I was allowed to use Maple (unless specifically told not to), calculators, the book, and all the material in the web-page of this class but **not** other resources on the internet.

I only spent (at most) 3 hours on doing the exam. The last 30 minutes were spent in checking and double-checking the answers.

I also understand that I may be subject to a random short chat to verify that I actually did it all by myself.

Signed:

Possibly useful facts from school Geometry (that you are welcome to use) : (i) The area of a circle radius r is πr^2. (ii) The circumference of a circle radius r is $2\pi r$ (iii) The parametric equation of an ellipse with axes a b and parallel to the x and y axes respectively is $x = a\cos\theta$, $y = b\cos\theta$, $0 < \theta < 2\pi$. (iv) The area of an ellipse with axes a and b is πab (v) The volume and surface area of a sphere radius R are $\frac{4}{3}\pi R^3$ and $4\pi R^2$ respectively (vi) The volume of a cone is the area of the base times the height over 3. (vii) The volume of a pyramid is the area of the base times the height over 3. (viii) The area of a triangle is base times height over 2.

Formula that you may (or may not) need

If the surface S is given in **explicit** notation $z = g(x, y)$, above the region of the xy-plane , D, then

$$\int\int_S \mathbf{F} \cdot d\mathbf{S} =$$

$$\int\int_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R \right) dA \quad .$$

1. (12 pts.) **Without using Maple (or any software)** Compute the **vector-field line integral**

$$\int_C (cos\,(e^{\sin x}) + 5y)\,dx \;+\; (sin\,(e^{\cos y}) \;+\; 11x\,)\,dy \quad ,$$

over the path consisiting of the five line segments (in that order)

$$(1,0) \to (-1,0) \to (-1,1) \to (0,2) \to (1,1) \to (1,0) \quad .$$

Explain! Clockwise (-1 x final answer)

---

ans. -30

Green's Theorem:

$P = \cos(e^{\sin x}) + 5y \qquad Q = \sin(e^{\cos y}) + 11x$

$\frac{\partial Q}{\partial x} = 11 \qquad \frac{\partial P}{\partial y} = 5$

$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 6$

$\iint_D 6 \cdot dA$ Since 6 is a constant, we can rewrite as $6 \cdot \iint_D dA$

The area is the sum of a rectangle and 2 right triangles:

$4 + \frac{1}{2} + \frac{1}{2} = 4 + 1 = 5$

$5 \cdot 6 = 30$ Since it is clockwise: $30 \cdot -1 = -30$

2. (12 points) Change the order of integration

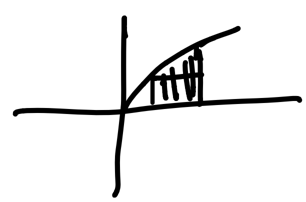
$$\int_{\frac{1}{4}}^{1} \int_{0}^{\sqrt{x}} f(x,y)\, \underbrace{dx\, dy}_{\text{typo?}} \quad dy\, dx$$

---

ans. $\displaystyle\int_0^{1/2}\int_{1/4}^{1} f(x,y)\,dx\,dy + \int_{1/2}^{1}\int_{y^2}^{1} f(x,y)\,dx\,dy$

$1/4 \le x \le 1$

$0 \le y \le \sqrt{x}$



New region:

R1: $0 \le y \le 1/2$

$1/4 \le x \le 1$

R2: $1/2 \le y \le 1$

$y^2 \le x \le 1$

This will be a sum of 2 integrals, one for the first rectangle, the second for the rest.

Final sum of integrals:

$$\int_0^{1/2}\int_{1/4}^{1} f(x,y)\,dx\,dy + \int_{1/2}^{1}\int_{y^2}^{1} f(x,y)\,dx\,dy$$

3. (12 points) Find the equation of the tangent plane at the point $(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6})$ to the surface given implicitly by

$$2\cos(x+y) + 4\cos(x+z) + 8\cos(y+z) = 7 \qquad .$$

Express you answer in **explicit** form, i.e in the format $z = ax + by + c$.

---

ans. $z = -x/2 - 5y/6 + 7\pi/18$

First, test the point:

$$2\cos\left(\frac{\pi}{6}+\frac{\pi}{6}\right) + 4\cos\left(\frac{\pi}{6}+\frac{\pi}{6}\right) + 8\cos\left(\frac{\pi}{6}+\frac{\pi}{6}\right)$$

$$= 2\cdot \tfrac{1}{2} + 4\cdot\tfrac{1}{2} + 8\cdot\tfrac{1}{2}$$

$$= 1 + 2 + 4 = 7 \checkmark$$

Let's say : $f = 2\cos(x+y) + 4\cos(x+z) + 8\cos(y+z) - 7$

$\frac{\partial f}{\partial x} = -2\sin(x+y) - 4\sin(x+z)$ at point: $-3\sqrt{3}$

$\frac{\partial f}{\partial y} = -2\sin(x+y) - 8\sin(y+z)$ at point: $-5\sqrt{3}$

$\frac{\partial f}{\partial z} = -4\sin(x+z) - 8\sin(y+z)$ at point: $-6\sqrt{3}$

$$-3\sqrt{3}(x-\pi/6) - 5\sqrt{3}(y-\pi/6) - 6\sqrt{3}(z-\pi/6) = 0$$

$$-3\sqrt{3}x + \frac{3\sqrt{3}\pi}{6} - 5y\sqrt{3} + \frac{5\sqrt{3}\pi}{6} - 6\sqrt{3}z + \frac{6\sqrt{3}\pi}{6} = 0$$

$$-3\sqrt{3}x - 5\sqrt{3}y - 6\sqrt{3}z + \frac{14\sqrt{3}}{6}\pi = 0$$

$$6\sqrt{3}z = -3\sqrt{3}x - 5\sqrt{3}y + \frac{14\sqrt{3}\pi}{6}$$

$$z = \frac{\left(-3\sqrt{3}x - 5\sqrt{3}y + \frac{14\sqrt{3}}{6}\pi\right)}{6\sqrt{3}}$$

Simplified, this is:

$$z = \frac{-x}{2} - \frac{5y}{6} + \frac{7\pi}{18}$$

4. (16 points) Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three vectors such that

$$\mathbf{a} \times \mathbf{b} = \mathbf{i} + \mathbf{j} - \mathbf{k} \quad , \quad \mathbf{b} \times \mathbf{c} = \mathbf{i} - \mathbf{j} + \mathbf{k} \quad , \quad \mathbf{a} \times \mathbf{c} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} \quad .$$

What is

$$(\mathbf{a} + \mathbf{b} + \mathbf{c}) \times (2\mathbf{a} - \mathbf{b} + 3\mathbf{c}) \qquad ?$$

---

ans. $3\hat{i} - 6\hat{j} + 9\hat{k}$

$(a+b+c) \times (2a - b + 3c)$

$(a \times 2a) + (a \times (-b)) + (a \times 3c)$

$0 + \langle -1, -1, 1 \rangle + \langle 6, 3, 6 \rangle = \langle 5, 2, 7 \rangle$

$+ (b \times 2a) + (b \times -b) + (b \times 3c)$

$-(2a \times b)$

$\langle -2, -2, 2 \rangle + 0 + \langle 3, -3, 3 \rangle = \langle 1, -5, 5 \rangle$

$+ (c \times 2a) + (c \times -b) + (c \times 3c)$

$-(2a \times c) \quad -(-b \times c)$

$\langle -4, -2, -4 \rangle + \langle 1, -1, 1 \rangle + 0 = \langle -3, -3, -3 \rangle$

$\langle 5, 2, 7 \rangle + \langle 1, -5, 5 \rangle + \langle -3, -3, -3 \rangle$

$= \langle 3, -6, 9 \rangle = 3\hat{i} - 6\hat{j} + 9\hat{k}$

5. (12 points) Find the three angles of the triangle ABC where

$$A = (0,0,0) \quad , \quad B = (1,0,1) \quad , \quad C = (1,1,0) \quad .$$

---

ans. The angle at A is: $\pi/3$ radians ;

The angle at B is: $\pi/3$ radians ;

The angle at C is: $\pi/3$ radians ;

$\angle A$:

$\overline{AB} = \langle 1,0,1 \rangle \qquad \overline{AC} = \langle 1,1,0 \rangle$

$AB \cdot AC = |AB|\,|BC| \cos\theta$

$1 = \sqrt{2} \cdot \sqrt{2} \cos\theta$

$1 = 2\cos\theta$

$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} = \angle A$

$\angle B$:

$\overline{BA} = \langle -1,0,-1 \rangle \qquad \overline{BC} = \langle 0,1,-1 \rangle$

$BA \cdot BC = |BA|\,|BC| \cos\theta$

$1 = 2\cos\theta$

$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} = \angle B$

$\angle C$:

$\overline{CA} = \langle -1,-1,0 \rangle \qquad \overline{CB} = \langle 0,-1,1 \rangle$

$CA \cdot CB = |CA|\,|CB| \cos\theta$

$1 = 2\cos\theta$

$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} = \angle B$

6. (12 points) Find the directional derivative of

$$f(x, y, z) = x^3 + y^3 + z^3 + xyz \quad ,$$

at the point $(1, 1, 1)$ in a direction pointing to the point $(-1, -1, -1)$.

---

ans. $-12/\sqrt{3}$

$\nabla f = \langle 3x^2 + yz,\ 3y^2 + xz,\ 3z^2 + xy \rangle$

$|\langle -1, -1, -1 \rangle| = \sqrt{1+1+1} = \sqrt{3}$

$u = \langle \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}} \rangle$

$\nabla f(1,1,1) = \langle 4, 4, 4 \rangle$

$\nabla f \cdot u = 4 \cdot \frac{-1}{\sqrt{3}} + 4 \cdot \frac{-1}{\sqrt{3}} + 4 \cdot \frac{-1}{\sqrt{3}}$

$= \frac{-4}{\sqrt{3}} - \frac{4}{\sqrt{3}} - \frac{4}{\sqrt{3}} = \boxed{\frac{-12}{\sqrt{3}}}$

7. (12 points) Using the Chain Rule (no credit for other methods), find

$$\frac{\partial g}{\partial u}$$

at $(u, v) = (0, 1)$, where

$$g(x, y) = 3x^2 - 3y^2 \quad ,$$

and

$$x = e^u \cos v \quad , \quad y = e^u \sin v \quad .$$

---

ans. $6\cos(1) - 6\sin(1)$

$$\frac{\partial g}{\partial x} = 6x \qquad \frac{\partial g}{\partial y} = -6y \qquad \frac{\partial x}{\partial u} = e^u \qquad \frac{\partial y}{\partial u} = e^u$$

$$\frac{\partial g}{\partial u} = \frac{\partial g}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial g}{\partial y} \cdot \frac{\partial y}{\partial u}$$

$$= (6x)(e^u) + (-6y)(e^u)$$

$$= (6 \cdot e^u \cos v)(e^u) + (-6 \cdot e^u \sin v)(e^u)$$

at $(u,v) = (0,1)$: $[6 \cdot e^0 \cos(1)](e^0) + [-6 \cdot e^0 \sin(1)](e^0)$

$$= (6\cos(1)) + (-6\sin(1))$$

$$= \boxed{6\cos(1) - 6\sin(1)} = 6(\cos(1) - \sin(1))$$

8. (12 points) Without using Maple (or any other software), compute the vector-field surface integral $\int_S \mathbf{F}.d\mathbf{S}$ if

$$\mathbf{F} = \langle\, 3x + cos(y^3 + yz)\,,\, -2y + e^{x+z^2}\,,\, 5z + \sin(xy^3 + e^x)\,\rangle \qquad ,$$

and S is the closed surface in 3D space bounding the region

$$\{(x, y, z) : x^2 + y^2 + z^2 < 4 \quad \textit{and} \quad \underline{x > 0} \quad \textit{and} \quad \underline{y < 0} \quad \textit{and} \quad \underline{z > 0}\} \quad .$$

---

ans. 8π

Divergence Theorem:

$$\operatorname{div} F = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z} = 3 + (-2) + 5 = 6$$

$$\iiint_E 6 \cdot dV = 6 \cdot \iiint_E \cdot\, dV$$

This is $6 \cdot$ Volume

Volume of sphere is $\frac{4}{3}\pi r^3$ where $r = 2$

but we have a restriction of $x > 0$, $y < 0$, $z > 0$, so I think we have to get only 1/8 of the volume.

So: $\frac{1}{8} \cdot \frac{4}{3}\pi r^3 = \frac{1}{6}\pi r^3 = \frac{\pi 8}{6} = \frac{4\pi}{3}$

Then multiply by 6. $\frac{6 \cdot 4\pi}{3} = 8\pi$

9. (12 points) Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if

$$\mathbf{F} = \langle\, 3z\,,\, 2x\,,\, y+z \,\rangle \quad ,$$

and S is the oriented surface

$$z = 2x + 3y \quad , \quad 0 < x < 1, \quad 0 < y < 1 \quad ,$$

with **upward pointing** normal.

---

ans. 15

$g = 2x+3y \qquad P = 3z \qquad Q = 2x \qquad R = y+z$

$\frac{\partial g}{\partial x} = 2 \qquad \frac{\partial g}{\partial y} = 3$

$\int\int_D (-3z)(2) - (2x)(3) + y + z$

$= \int\int_D -6z - 6x + y + z = \int\int_D -6(2x+3y) - 6x + y + 2x+3y$

$= \int_0^1 \int_0^1 -6(2x+3y) - 6x + y + 2x + 3y \, dx\, dy$

Using maple, this is 15.

10. (12 points) Without using Maple or software, find the critical point(s) of

$$f(x, y) = 4x - y^2 - \ln(2x + y) \quad ,$$

and decide for each whether it is a local maximum, local minimum, or saddle point. Explain.

---

ans. $(3/4, -1)$ is a saddle point

$f_x = 4 - \frac{2}{2x+y}$ $\qquad f_y = -2y - \frac{1}{2x+y}$

$f_{xx} = \frac{4}{(2x+y)^2}$ $\qquad f_{xy} = \frac{2}{(2x+y)^2}$ $\qquad f_{yy} = -2 + \frac{1}{(2x+y)^2}$

$4 - \frac{2}{2x+y} = 0$ $\qquad -2y - \frac{1}{2x+y} = 0$

Critical Point: $(3/4, -1)$

$D = f_{xx} f_{yy} - [f_{xy}]^2$

$D = 16 \cdot 2 - (8)^2$

$= 32 - 64 = -32 < 0$

Since the discriminent is < 0,
$(3/4, -1)$ is a saddle point

11. (12 points) Without using Maple or software, using a **Linearization** around the point $(1, 1, 2)$, approximate $f(1.001, 0.999, 2.001)$ if

$$f(x, y, z) = \sqrt{2x^2 + 3y^2 + z^2} \quad .$$

---

ans. 9001/3000

$$f(x,y,z) = (2x^2 + 3y^2 + z^2)^{1/2}$$

$$f_x = \frac{1}{2} \cdot \frac{4x}{\sqrt{2x^2+3y^2+z^2}} = \frac{2x}{\sqrt{2x^2+3y^2+z^2}}$$

$$f_y = \frac{1}{2} \cdot \frac{6y}{\sqrt{2x^2+3y^2+z^2}} = \frac{3y}{\sqrt{2x^2+3y^2+z^2}}$$

$$f_z = \frac{1}{2} \cdot \frac{2z}{\sqrt{2x^2+3y^2+z^2}} = \frac{z}{\sqrt{2x^2+3y^2+z^2}}$$

$f(1,1,2) = 3 \qquad f_x(1,1,2) = \frac{2}{3} \qquad f_y(1,1,2) = 1$

$f_z(1,1,2) = 2/3$

$$L(x,y,z) = 3 + 2/3(1.001-1) + 1(0.999-1) + 2/3(2.001-2)$$

$$= 3 + 2/3(0.001) + (-0.001) + 2/3(0.001)$$

$$= 3 + \frac{2}{3} \cdot \frac{1}{1000} - \frac{1}{1000} + \frac{2}{3} \cdot \frac{1}{1000}$$

$$= 3 + \frac{2}{3000} - \frac{3}{3000} + \frac{2}{3000}$$

$$= \frac{9000}{3000} + \frac{2}{3000} - \frac{3}{3000} + \frac{2}{3000} = \frac{9001}{3000}$$

12. (12 points) Without using Maple (or any other software) and by using polar coordinates (no credit for doing it directly) find

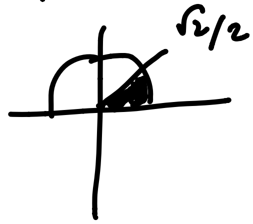
$$\int_0^{\frac{\sqrt{2}}{2}} \int_0^x x\,dy\,dx + \int_{\frac{\sqrt{2}}{2}}^1 \int_0^{\sqrt{1-x^2}} x\,dy\,dx \quad .$$

Explain!

---

ans. $\sqrt{2}/6$

If we graph this, we see a figure similar to



Using polar coordinates: $0 \le r \le 1$

$0 \le \theta \le \pi/4$

$x = r\cos\theta$

$$\int_0^{\pi/4}\int_0^1 r\cos\theta \, r\,dr\,d\theta = \int_0^{\pi/4}\int_0^1 r^2\cos\theta\,dr\,d\theta$$

$$\to \int_0^1 r^2\cos\theta\,dr = \frac{r^3}{3}\cos\theta\Big|_0^1 = \frac{1}{3}\cos\theta$$

$$\frac{1}{3}\cdot\int_0^{\pi/4}\cos\theta\,d\theta = \sin\theta\Big|_0^{\pi/4} = \frac{\sqrt{2}}{2}\cdot\frac{1}{3} = \boxed{\frac{\sqrt{2}}{6}}$$

13. (12 points) Convert the triple iterated integral

$$\int_0^{2} \int_0^{\sqrt{4-z^2}} \int_{-\sqrt{4-z^2-y^2}}^{0} x^2\, y\, z\, dx\, dy\, dz$$

to **spherical coordinates**. **Do not evaluate.**

---

ans. $\int_0^{\pi/2} \int_{\pi/2}^{\pi} \int_0^{2} (\rho \sin\phi \cos\theta)^2 (\rho \sin\phi \sin\theta)(\rho \cos\phi)\rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$

From the problem, it seems that

$x < 0, \quad y > 0, \quad z > 0$

Therefore:

$0 \leq \rho \leq 2, \quad \pi/2 \leq \theta \leq \pi, \quad 0 \leq \phi \leq \pi/2$

$x = \rho \sin\phi \cos\theta \qquad y = \rho \sin\phi \sin\theta \qquad z = \rho \cos\phi$

$dV = \rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$

$$\int_0^{\pi/2} \int_{\pi/2}^{\pi} \int_0^{2} (\rho \sin\phi \cos\theta)^2 (\rho \sin\phi \sin\theta)(\rho \cos\phi)\rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$$

Sorry it's so messy :(

14. (12 points) Find the curvature of the curve

$$\mathbf{r}(t) = \langle 5, 3\sin t, 3\cos t\rangle$$

at the point where $t = \frac{\pi}{3}$.

---

ans. $1/3$

$r'(t) = \langle 0, 3\cos t, -3\sin t \rangle$

$r''(t) = \langle 0, -3\sin t, -3\cos t \rangle$

$r'(t) \times r''(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3\cos t & -3\sin t \\ 0 & -3\sin t & -3\cos t \end{vmatrix}$

$\hat{i}(-9\cos^2 t - 9\sin^2 t) - \hat{j}(0-0) + \hat{k}(0-0)$

$= \langle -9\cos^2 t - 9\sin^2 t, 0, 0 \rangle = \langle -9(\sin^2 t + \cos^2 t), 0, 0 \rangle$

$= \langle -9, 0, 0 \rangle$

$|r'(t) \times r''(t)| = \sqrt{81+0+0} = 9$

$|r'(t)| = \sqrt{9\cos^2 t + 9\sin^2 t} = \sqrt{9(1)} = 3$

$k(t) = \frac{9}{3^3} = \frac{9}{27} = \boxed{\frac{1}{3}}$

15. (12 points) Set-up an iterated double integral, in type I format, but do not compute, for the surface area of the surface given parameterically by

$$\mathbf{r}(u,v) = \langle u^2 , uv , v^2 \rangle \quad , \quad 0 < u < v < 1 \quad .$$

---

ans. $\displaystyle\int_0^1 \int_0^v 2\sqrt{v^4 + 4u^2v^2 + u^4}\, du\, dv$

Surface is described parametrically

$r_u = \langle 2u, v, 0 \rangle$ $\qquad$ $0 < u < v$

$r_v = \langle 0, u, 2v \rangle$ $\qquad$ $0 < v < 1$

$dS = |r_u \times r_v|\, du\, dv$

$$r_u \times r_v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2u & v & 0 \\ 0 & u & 2v \end{vmatrix}$$

$\hat{i}(2v^2 - 0) - \hat{j}(4uv - 0) + \hat{k}(2u^2 - 0)$

$\langle 2v^2, 4uv, 2u^2 \rangle$

$|r_u \times r_v| = \sqrt{4v^4 + 16u^2v^2 + 4u^4} = \sqrt{4(v^4 + 4u^2v^2 + u^4)}$

$dS = 2\sqrt{v^4 + 4u^2v^2 + u^4}\, du\, dv$

$\therefore \displaystyle\int_0^1 \int_0^v 2\sqrt{v^4 + 4u^2v^2 + u^4}\, du\, dv$

16. (12 points) Let

$$f(x, y, z) = xy^2z^3 \quad ,$$

and let

$$g(x, y, z) = x + y^2 + z^3 \quad .$$

compute the dot-product

$$grad(f) \, . \, grad(g) \qquad .$$

at the point $(1, 1, 1)$.

---

ans. 14

$\nabla f = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$

$\nabla g = \langle 1, 2y, 3z^2 \rangle$

$\nabla f(1,1,1) = \langle 1, 2, 3 \rangle$

$\nabla g(1,1,1) = \langle 1, 2, 3 \rangle$

$\langle 1,2,3 \rangle \cdot \langle 1,2,3 \rangle = 1 + 4 + 9 = \boxed{14}$

17. (8 points) Decide whether the following limit exists. If it does ,find it. If it does not, explain why it does not exist.

$$\lim_{(x,y,z,w)\to(0,0,0,0)} \frac{(x+y)^2-(z+w)^2}{x+y-z-w} \quad .$$

---

ans. 0

Using Maple, I saw that you can simplify the expression to:

$w+z+x+y$

Now:

$$\lim_{(x,y,z,w)\to 0} w+z+x+y = 0+0+0+0 = 0$$

Therefore, the limit exists and it is 0.