NAME: (print!) AASTHA KASERA RUID: (print!) 193006075

SSC: (circle) None / I / II / I and II

MATH 251 (22,23,24) [Fall 2020], Dr. Z. , Final Exam , Tue., Dec. 15, 2020

Email the completed test, renamed as `finalFirstLast.pdf` to DrZcalc3@gmail.com no later than 3:30pm, (or, in case of conflict, three and half hours after the start).

WRITE YOUR FINAL ANSWERS BELOW

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1. 18
2. $\int_0 \int_{y^2}^{1} f(x,y)\,dx\,dy$
3. $-3\sqrt{3}x - 5\sqrt{3}y + \frac{8\sqrt{3}\pi + \pi}{6} = z$
4. $3\hat{i} - 6\hat{j} + 9\hat{k}$
5. $\pi/3,\ \pi/4,\ 5\pi/12$
6. $-24\sqrt{12}$
7. $6(\cos^2 1 - \sin^2 1)$
8. $4\pi/3$
9. -15
10. $(\frac{3}{4}, -1) \rightarrow$ Saddle Point
11. 3.0003
12. $\sqrt{2}/6$
13.
14. $\frac{1}{3}$
15. $\longrightarrow$
16. 14
17. 0

13. $\int_{\pi/2}^{\pi}\int_0^{\pi/2}\int_0^{2} \rho^6 \sin^4 \phi \cos^2\theta \sin\theta \cos\phi \, d\rho\, d\phi\, d\theta$

15. $\int_0^1 \int_0^1 \sqrt{4v^4 + 16u^2v^2 + 4u^4}\, dv\, du$

Sign the following declaration:

I Hereby declare that all the work was done by myself. I was allowed to use Maple (unless specifically told not to), calculators, the book, and all the material in the web-page of this class but **not** other resources on the internet.

I only spent (at most) 3 hours on doing the exam. The last 30 minutes were spent in checking and double-checking the answers.

I also understand that I may be subject to a random short chat to verify that I actually did it all by myself.

Signed:

Possibly useful facts from school Geometry (that you are welcome to use) : (i) The area of a circle radius r is πr^2. (ii) The circumference of a circle radius r is $2\pi r$ (iii) The parametric equation of an ellipse with axes a b and parallel to the x and y axes respectively is $x = a\cos\theta$, $y = b\cos\theta$, $0 < \theta < 2\pi$. (iv) The area of an ellipse with axes a and b is πab (v) The volume and surface area of a sphere radius R are $\frac{4}{3}\pi R^3$ and $4\pi R^2$ respectively (vi) The volume of a cone is the area of the base times the height over 3. (vii) The volume of a pyramid is the area of the base times the height over 3. (viii) The area of a triangle is base times height over 2.

Formula that you may (or may not) need

If the surface S is given in **explicit** notation $z = g(x, y)$, above the region of the xy-plane , D, then

$$\int\int_S \mathbf{F} \cdot d\mathbf{S} =$$

$$\int\int_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right)\, dA \quad .$$

1. (12 pts.) **Without using Maple (or any software)** Compute the **vector-field line integral**

$$\int_C (cos\,(e^{\sin x}) + 5y)\,dx \;+\; (sin\,(e^{\cos y}) \;+\; 11x\,)\,dy \quad ,$$

over the path consisiting of the five line segments (in that order)

$$(1,0) \to (-1,0) \to (-1,1) \to (0,2) \to (1,1) \to (1,0) \quad .$$

Explain!

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ans. Number - 18

Using Green's theorem:

the function goes counterclockwise

$P = \cos(e^{\sin x}) + 5y$

$Q = \sin(e^{\cos y}) + 11x$

$Q_x = 11$

$P_y = 5$

$\iint Q_x - P_y \, dA = \iint 11-5 \, dA$

$= 6 \iint dA$

$= 6 \cdot \text{Area}$

$\frac{2(b_1+b_2)h}{2}$

$= 2\frac{(2+1)}{2}1 = 3$

$= 6 \times 3 = 18$

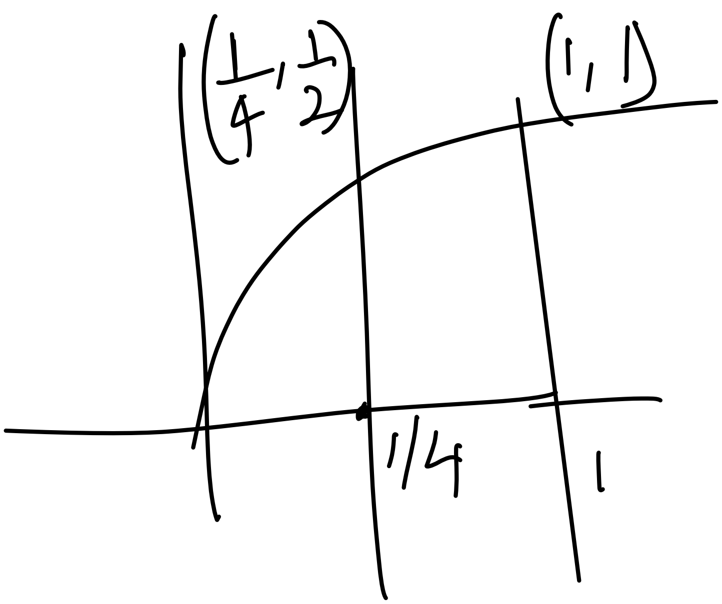
2. (12 points) Change the order of integration

$$\cancel{\int_{\frac{1}{4}}^{1} \int_{0}^{\sqrt{x}} f(x, y)\, dx\, dy}\ . \qquad \int_{1/4}^{1} \int_{0}^{\sqrt{x}} f(x,y)\, dy\, dx$$

ans. $\displaystyle \int_{0}^{1} \int_{y^2}^{1} f(x,y)\, dx\, dy$

$0 \le y \le \sqrt{x} \qquad 1/4 \le x \le 1$

$0 \le y \le 1 \quad y^2 \le x \le 1$



$$\int_{0}^{1} \int_{y^2}^{1} f(x,y)\, dx\, dy$$

3. (12 points) Find the equation of the tangent plane at the point $(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6})$ to the surface given implicitly by

$$2\cos(x+y) + 4\cos(x+z) + 8\cos(y+z) = 7 \qquad .$$

Express you answer in **explicit** form, i.e in the format $z = ax + by + c$.

ans. $z = -3\sqrt{3}x - 5\sqrt{3}y + \frac{8\sqrt{3}\pi + \pi}{6}$

$= 2\cos\left(\frac{\pi}{3}\right) + 4\cos\left(\frac{\pi}{3}\right) + 8\cos\left(\frac{\pi}{3}\right)$

$= 2\left(\frac{1}{2}\right) + 4\left(\frac{1}{2}\right) + 8\left(\frac{1}{2}\right)$

$= 1 + 2 + 4$

$= 7$ (so it lies on the plane)

$f'(x)(x-x_0) + f'(y)(y-y_0) = z - z_0$

$f_x(x,y,z) = -2\sin(x+y) - 4\sin(x+z)$

$f_y(x,y,z) = -2\sin(x+y) - 8\sin(y+z)$

At $\left(\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}\right)$

$f_x = -2\frac{\sqrt{3}}{2} - 4\frac{\sqrt{3}}{2} = -\sqrt{3} - 2\sqrt{3} = -3\sqrt{3}$

$f_y = -2\frac{\sqrt{3}}{2} - 8\frac{\sqrt{3}}{2} = -\sqrt{3} - 4\sqrt{3} = -5\sqrt{3}$

$$-3\sqrt{3}\left(x-\frac{\pi}{6}\right)-5\sqrt{3}\left(y-\frac{\pi}{6}\right)=2-\frac{\pi}{6}$$

$$-3\sqrt{3}x+\frac{3\sqrt{3}\pi}{6}-5\sqrt{3}y+\frac{5\sqrt{3}\pi}{6}+\frac{\pi}{6}=2$$

$$=-3\sqrt{3}x-5\sqrt{3}y+\frac{3\sqrt{3}}{}\pi+\frac{5\sqrt{3}}{6}\pi+\frac{\pi}{6}=2$$

$$-3\sqrt{3}x-5\sqrt{3}y+\frac{8\sqrt{3}\pi+\pi}{6}=2$$

4. (16 points) Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three vectors such that

$$\mathbf{a} \times \mathbf{b} = \mathbf{i} + \mathbf{j} - \mathbf{k} \quad , \quad \mathbf{b} \times \mathbf{c} = \mathbf{i} - \mathbf{j} + \mathbf{k} \quad , \quad \mathbf{a} \times \mathbf{c} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} \quad .$$

What is

$$(\mathbf{a} + \mathbf{b} + \mathbf{c}) \times (2\mathbf{a} - \mathbf{b} + 3\mathbf{c}) \qquad ?$$

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ans: $3\hat{i} - 6\hat{j} + 9\hat{k}$

Using the theorem:

$$\Big[\underbrace{(a \times 2a)}_{=0} + (a \times -b) + (a \times 3c)\Big] \times$$

$$\Big[(b \times 2a) + \underbrace{(b \times -b)}_{=0} + (b \times 3c)\Big] \times$$

$$\Big[(c \times 2a) + (c \times -b) + \underbrace{(c \times 3c)}_{=0}\Big]$$

$$= \Big[(-\hat{i} - \hat{j} + \hat{k}) + (6\hat{i} + 3\hat{j} + 6\hat{k})\Big] +$$

$$\Big[(-2\hat{i} - 2\hat{j} + 2\hat{k}) + (3\hat{i} - 3\hat{j} + 3\hat{k})\Big] +$$

$$\Big[(-4\hat{i} - 2\hat{j} - 4\hat{k}) + (\hat{i} - \hat{j} + \hat{k})\Big] +$$

$$= \Big[(5\hat{i} + 2\hat{j} + 7\hat{k}) + (1\hat{i} - 5\hat{j} + 5\hat{k}) +$$

$$(-3\hat{i} - 3\hat{j} - 3\hat{k})\Big]$$

$$= 3\hat{i} - 6\hat{j} + 9\hat{k}$$

5. (12 points) Find the three angles of the triangle ABC where

$$A = (0,0,0) \quad , \quad B = (1,0,1) \quad , \quad C = (1,1,0) \quad .$$

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ans. The angle at A is: radians ; $\pi/3$

The angle at B is: radians ; $\pi/4$

The angle at C is: radians ; $5\pi/12$

$A = (0,0,0)$ $\qquad$ $AB = (1, 0, 1)$

$B = (1,0,1)$

$C = (1,1,0)$ $\qquad$ $AC = (1, 1, 0)$

$$\text{Angle at } A = \frac{(1)\cdot(1) + (0)(1) + 1(0)}{\sqrt{(1^2+1^2)} \cdot \sqrt{1^2+1^2}}$$

$$\cos A = \frac{1}{\sqrt{2}\cdot\sqrt{2}}$$

$$\cos A = \frac{1}{2}$$

$$A = 60^\circ = \frac{\pi}{3}$$

Angle at $B =$

$BA = (-1, 0, -1)$

$BC = (0, 1, -1)$

$$\cos B = \frac{(-1) + (0) - 1}{2}$$

$$\cos B = \frac{1}{\sqrt{2}}$$

$$B = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

Since the sum of angles $= \pi$

$$\frac{\pi}{4} + \frac{\pi}{3} + x = \pi$$

$$x = \pi - \frac{\pi}{4} - \frac{\pi}{3}$$

$$= \pi - \left(\frac{3\pi + 4\pi}{12}\right)$$

$$= \pi - \frac{7\pi}{12}$$

$$= \frac{5\pi}{12}$$

6. (12 points) Find the directional derivative of

$$f(x, y, z) = x^3 + y^3 + z^3 + xyz \quad ,$$

at the point $(1, 1, 1)$ in a direction pointing to the point $(-1, -1, -1)$.

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ans. $-24/\sqrt{12}$

$\nabla f(x,y,z) = \langle 3x^2 + yz,\ 3y^2 + xz,\ 3z^2 + xy \rangle$

$(1, 1, 1) \rightarrow (-1, -1, -1)$

$\nabla f(x,y,z) = \langle 4, 4, 4 \rangle \quad \langle -2, -2, -2 \rangle$

In the direction $(-1, -1, -1)$

Unit direction is

$$\left\langle \frac{-2}{\sqrt{12}}, \frac{-2}{\sqrt{12}}, \frac{-2}{\sqrt{12}} \right\rangle$$

directional vector =

$$\langle 4, 4, 4 \rangle \cdot \left\langle \frac{-2}{\sqrt{12}}, \frac{-2}{\sqrt{12}}, \frac{-2}{\sqrt{12}} \right\rangle$$

$$= -\frac{8}{\sqrt{12}} - \frac{8}{\sqrt{12}} - \frac{8}{\sqrt{12}}$$

$$= -\frac{24}{\sqrt{12}}$$

7. (12 points) Using the Chain Rule (no credit for other methods), find

$$\frac{\partial g}{\partial u}$$

at $(u, v) = (0, 1)$, where

$$g(x, y) = 3x^2 - 3y^2 \quad ,$$

and

$$x = e^u \cos v \quad , \quad y = e^u \sin v \quad .$$

ans. $6(\cos^2 1 - \sin^2 1)$

$$\frac{dg}{du} = \frac{dg}{dx} \cdot \frac{dx}{du} + \frac{dg}{dy} + \frac{dy}{du}$$

$$= \left(6x \cdot e^u \cos v\right) + \left(-6y \cdot e^u \cos v\right)$$

$$= 6x e^u \cos v - 6y e^u \sin v$$

$$= 6e^u (x \cos v - y \sin v)$$

At $(u, v) = (0, 1)$

$$x = e^0 \cos 1$$

$$y = e^0 \sin 1$$

$$\frac{dg}{du} = 6e^0 (\cos 1 \cdot \cos 1 - \sin 1 \cdot \sin 1)$$

$$= 6(\cos^2 1 - \sin^2 1)$$

8. (12 points) Without using Maple (or any other software), compute the vector-field surface integral $\int_S \mathbf{F}.d\mathbf{S}$ if

$$\mathbf{F} = \langle\, 3x + cos(y^3 + yz)\,,\, -2y + e^{x+z^2}\,,\, 5z + \sin(xy^3 + e^x)\,\rangle \qquad ,$$

and S is the closed surface in 3D space bounding the region

$$\{(x, y, z) : x^2 + y^2 + z^2 < 4 \quad and \quad x > 0 \quad and \quad y < 0 \quad and \quad z > 0\} \quad .$$

---

ans. $4\pi/3$

Divergence theorem:

$$div(F) = 3 - 2 + 5$$
$$= 1 + 5$$
$$= 6.$$

$$= 6\iiint dV.$$

$$= 6\int_{3\pi/2}^{2\pi}\int_0^{\pi/2}\int_0^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$\int_{3\pi/2}^{2\pi}\int_{0}^{\pi/2}\left[\frac{\rho^3 \sin\phi}{3}\right]\Bigg|_0^2 \, d\phi \, d\theta$$

1x

$$\int_{3\pi/2}^{2\pi} -\frac{8\cos\phi}{3}\Bigg|_0^{\pi/2} \, d\theta$$

$$\left[\frac{8}{3}\phi\right]\Bigg|_{3\pi/2}^{2\pi}$$

$$= \frac{4\pi}{3}$$

9. (12 points) Compute the vector-field surface integral $\int\int_S \mathbf{F}.d\mathbf{S}$ if

$$\mathbf{F} = \langle 3z \, , \, 2x \, , \, y+z \rangle \quad ,$$

and S is the oriented surface

$$z = 2x + 3y \quad , \quad 0 < x < 1, \quad 0 < y < 1 \quad ,$$

with **upward pointing** normal.

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ans. -15

$P = 3z$

$Q = 2x$

$R = y + z$

$g(x,y) = 2x + 3y$

$$\int\int_S \mathbf{F} \cdot d\mathbf{S} =$$

$$\int\int_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R \right) dA \quad .$$

$$\int\int -3z(2) - 2x(3) + y + z \quad dx\,dy$$

$$\int_0^1\int_0^1 -6z - 6x + y + z \quad dx\,dy$$

$$\int_0^1\int_0^1 -6(2x+3y) - 6x \quad + y + 2x + 3y$$

$$\int_0^1\int_0^1 -12x - \underline{18y} \quad -6x \quad +\underline{y} + 2x + \underline{3y} \, dx\,dy$$

$$\int_0^1\int_0^1 -16x - 14y \, dx\,dy$$

$$= -15$$

10. (12 points) Without using Maple or software, find the critical point(s) of

$$f(x, y) = 4x - y^2 - \ln(2x + y) \quad ,$$

and decide for each whether it is a local maximum, local minimum, or saddle point. Explain.

ans. $\left(\frac{3}{4}, -1\right) \to$ saddle point

$$f(x,y) = 4x - y^2 - \ln(2x+y)$$

$$f_x = 4 - \frac{2}{2x+y}$$

$$f_y = -2y - \frac{1}{2x+y}$$

$$4 = \frac{2}{2x+y} \qquad \underline{8x + 4y = 2}$$

$$-2y = \frac{1}{2x+y}.$$

$$-4xy - 2y^2 = 1$$

$$-2y(2x - y) = 1$$

$$\boxed{x = \frac{3}{4} \quad y = -1}$$

At this point

$$f_{xx} = \frac{4}{(2x+y)^2}$$

$$f_{yy} = -2 + \frac{1}{(2x+y)^2}$$

$$f_{xy} = \frac{2}{(2x+y)^2}$$

$$f_x = 4 - \frac{2}{2x+y}$$

$$f_y = -2y - \frac{1}{2x+y}$$

$$f_{xx} = \frac{4}{\left(2\left(\frac{3}{4}\right) - 1\right)^2} = \frac{4}{\left(\frac{3}{2} - 1\right)^2} = \frac{4}{\left(\frac{1}{2}\right)^2}$$

$$= \frac{4}{\frac{1}{4}}$$

$$= 4 \cdot 4$$

$$= 16$$

$$f_{yy} = -2 + 4$$

$$= 2.$$

$$f_{xy} = 2 \times 4 = 8$$

$$D = 2 \cdot 16 - (64) \quad D < 0$$

$\therefore$ Saddle point

11. (12 points) Without using Maple or software, using a **Linearization** around the point $(1, 1, 2)$, approximate $f(1.001, 0.999, 2.001)$ if

$$f(x, y, z) = \sqrt{2x^2 + 3y^2 + z^2} \quad .$$

---

ans. 3.0003

$f(x,y,z) = \sqrt{2x^2+3y^2+z^2}$ At $(1,1,2)$

$f_x = \dfrac{4x}{2\sqrt{2x^2+3y^2+z^2}}$ $f_x = 4$

$f_y = \dfrac{6y}{2\sqrt{2x^2+3y^2+z^2}}$

$f_z = \dfrac{2z}{2\sqrt{2x^2+3y^2+z^2}}$

$f(1,1,2) = \sqrt{2+3+4}$
$= \sqrt{9} = 3$

At (1, 1, 2)

$$f_x = \frac{4}{2\sqrt{2+3+4}}$$

$$= \frac{2}{\sqrt{9}} = \frac{2}{3}$$

$$f_y = \frac{6}{2\sqrt{9}} = 1$$

$$f_z = \frac{4}{2\sqrt{9}} = \frac{2}{3}$$

$$\text{linearization} = 3 + \frac{2}{3}(1.001 - 1) + 1(0.999 - 1) + \frac{2}{3}(2.001 - 2)$$

$$= \underline{\underline{3.00}}03$$

12. (12 points) Without using Maple (or any other software) and by using polar coordinates (no credit for doing it directly) find

$$\int_0^{\frac{\sqrt{2}}{2}} \int_0^{x} x\,dy\,dx + \int_{\frac{\sqrt{2}}{2}}^{1} \int_0^{\sqrt{1-x^2}} x\,dy\,dx \quad .$$

Explain!

---

ans. $\sqrt{2}/6$

$$\int_0^{\frac{\sqrt{2}}{2}} \int_0^{x} x\,dy\,dx$$

$$\int_0^{\sqrt{2}/2} xy \Big|_0^{x} dx$$

$$= \int_0^{\sqrt{2}/2} x^2\,dx$$

$$\frac{x^3}{3}\Big|_0^{\sqrt{2}/2}$$

$$= \frac{\left(\frac{\sqrt{2}}{2}\right)^3}{3} = \frac{\frac{2\sqrt{2}}{8}}{3} = \frac{\sqrt{2}}{12}$$

$$\int_0^{\pi/2}\int_0^{\sqrt{2}/2} r^2 \cos\phi \, dr \, d\phi =$$

$$\int_0^{\pi/2} \left[\frac{r^3}{3} \cos\theta\right]_0^{\sqrt{2}/2}$$

$$\int_0^{\pi/2} \frac{\sqrt{2}}{12} \cos\theta \, d\theta$$

$$\left[\frac{\sqrt{2}}{12} \sin\theta\right]_0^{\pi/2}$$

$$= \frac{\sqrt{2}}{12}$$

$$\frac{\sqrt{2}}{12} + \frac{\sqrt{2}}{12} = \frac{\sqrt{2}}{6}$$

13. (12 points) Convert the triple iterated integral

$$\int_0^4 \int_0^{\sqrt{4-z^2}} \int_{-\sqrt{4-z^2-y^2}}^{0} x^2\, y\, z\, dx\, dy\, dz$$

to **spherical coordinates**. **Do not evaluate.**

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ans. $\int_{\pi/2}^{\pi}\int_0^{\pi/2}\int_0^{2} \rho^6 \sin^4\phi \cos^2\theta \sin\theta \cos\phi \; d\rho\, d\phi\, d\theta$

$-\sqrt{4-z^2-y^2} \le x \le 0$

$0 \le y \le \sqrt{4-z^2}$

$0 \le z \le 4$

$$\int_{\pi/2}^{\pi}\int_0^{\pi/2}\int_0^{2} (\rho^2 \sin^2\phi \cos^2\theta)(\rho \sin\phi \sin\theta)(\rho\cos\phi)\, \rho^2 \sin\phi \; d\rho\, d\phi\, d\theta$$

$$\int_{\pi/2}^{\pi}\int_0^{\pi/2}\int_0^{2} \rho^6 \sin^4\phi \cos^2\theta \sin\theta \cos\phi \; d\rho\, d\phi\, d\theta$$

14. (12 points) Find the curvature of the curve

$$\mathbf{r}(t) = \langle 5, 3\sin t, 3\cos t\rangle$$

at the point where $t = \frac{\pi}{3}$.

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ans. $1/3$

$\underline{\text{At } t = \pi/3}$

$r(t) = \langle 5, 3\sin t, 3\cos t\rangle$

$r'(t) = \langle 0, 3\cos t, -3\sin t\rangle$

$r''(t) = \langle 0, -3\sin t, -3\cos t\rangle$

$|r'(t)| = \sqrt{3^2\cos^2 t + 3^2\sin^2 t}$

$= \sqrt{9}$

$= 3$

$\text{Curvature} = \dfrac{|r'(t) \times r''(t)|}{|r'(t)|^3}$

$r'(t) = \langle 0, \frac{3}{2}, -\frac{3\sqrt{3}}{2}\rangle$

$r''(t) = \langle 0, -\frac{3\sqrt{3}}{2}, -\frac{3}{2}\rangle$

$$r'(t) \times r''(t) = \begin{array}{c} \begin{array}{ccc} i & j & k \end{array} \\ \begin{vmatrix} 0 & \frac{3}{2} & -\frac{3\sqrt{3}}{2} \\ 0 & -\frac{3\sqrt{3}}{2} & -\frac{3}{2} \end{vmatrix} \end{array}$$

$$\left(-\frac{9}{4} - \left(\frac{27}{4}\right)\right)\hat{i} - 0\hat{j} + 0\hat{k}$$

$$= -\frac{9}{4} - \frac{27}{4}$$

$$= -\frac{36}{4}$$

$$= -9$$

$$= \frac{|-9|}{|r'(t)|^3}$$

$$r'(t) = \sqrt{0 + \left(\frac{3}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2}$$

$$= \sqrt{\frac{9}{4} + \frac{27}{4}}$$

$$= \sqrt{\frac{36}{4}}$$

$$= \sqrt{9}$$

$$= 3$$

$$\text{Curvature} = \frac{9}{(3)^3}$$

$$= \frac{9}{9 \cdot 3} = \frac{1}{3}$$

15. (12 points) Set-up an iterated double integral, in type I format, but do not compute, for the surface area of the surface given parameterically by

$$\mathbf{r}(u,v) = \langle u^2 \,, uv \,, v^2 \rangle \quad , \quad 0 < u < v < 1 \quad .$$

---

ans.

$$\boxed{\begin{array}{l} 0 < u < 1 \\ 0 < v < 1 \end{array}}$$

$r\_u = \langle 2u, v, 0 \rangle$

$r\_v = \langle 0, u, 2v \rangle$ d

$r\_u \times r\_v$

$$\begin{vmatrix} i & j & k \\ \underline{2u} & v & 0 \\ 0 & u & 2v \end{vmatrix}$$

$$\int_0^1 \int_{\cancel{0}}^1 dS$$

$$\underline{2v^2 \hat{i} - 4uv \hat{j} + 2u^2 \hat{k}}$$

$$= \int_0^1 \int_u^1 \sqrt{4v^4 + 16u^2v^2 + 4u^4}\, dv\, du$$

$x = u^2 \qquad y = uv \qquad z = v^2$

16. (12 points) Let

$$f(x,y,z) \;=\; xy^2z^3 \quad ,$$

and let

$$g(x,y,z) \;=\; x+y^2+z^3 \quad .$$

compute the dot-product

$$grad(f)\,.\,grad(g) \qquad .$$

at the point $(1,1,1)$.

---

ans. 14

$\nabla f(x,y,z) = \langle y^2z^3, 2xyz^3, 3z^2xy^2 \rangle$

$\nabla g(x,y,z) = \langle 1, 2y, 3z^2 \rangle$

At $(1,1,1)$

$\nabla f \cdot \nabla f = \langle 1, 2, 3 \rangle \langle 1, 2, 3 \rangle$

$= 1+4+9$

$= 14$

17. (8 points) Decide whether the following limit exists. If it does ,find it. If it does not, explain why it does not exist.

$$\lim_{(x,y,z,w)\to(0,0,0,0)} \frac{(x+y)^2-(z+w)^2}{x+y-z-w} \quad .$$

---

ans. 0

$y = ax$

$z = bx$

$w = 0x$

$(x+y)^2 - (z-w)^2$

$(x+y+z+w)(\cancel{x}+y-z-w)$

$z+y+x+w$

$$\lim_{(x,y,z,w)\to(0,0,0,0)}$$

$= 0$