Solutions to the "QUIZ" for SEPT. 3, 2009

1. Show that the triangle with vertices $P = (1, 0, 0)$, $Q = (0, 1, 0)$, and $R = (0, 0, 1)$ is an equilateral triangle.

Solution:

$$dist(P, Q) = \sqrt{(1-0)^2 + (0-1)^2 + (0-0)^2} = \sqrt{2} \quad ,$$

$$dist(P, R) = \sqrt{(1-0)^2 + (0-0)^2 + (0-1)^2} = \sqrt{2} \quad ,$$

$$dist(Q, R) = \sqrt{(0-0)^2 + (0-1)^2 + (1-0)^2} = \sqrt{2} \quad .$$

Since the three sides of triangle PQR all have the same lengths, it follows that the triangle is equilateral.

2. Determine whether the following two lines ever meet. If they do meet, where?

$$\mathbf{r}_1(t) = \langle 1, 0, 0 \rangle + t\langle 1, 2, 3 \rangle \quad , \quad \mathbf{r}_2(t) = \langle 0, 1, 0 \rangle + t\langle 2, 1, 3 \rangle \quad .$$

Solution: The very **first** step is to use another symbol, say s, for $\mathbf{r}_2(t) = \langle 0, 1, 0 \rangle + t\langle 2, 1, 3 \rangle$, getting

$$\mathbf{r}_1(t) = \langle 1, 0, 0 \rangle + t\langle 1, 2, 3 \rangle \quad , \quad \mathbf{r}_2(s) = \langle 0, 1, 0 \rangle + s\langle 2, 1, 3 \rangle \quad .$$

Spelling it out, we have

$$\mathbf{r}_1(t) = \langle 1 + t, 2t, 3t \rangle \quad , \quad \mathbf{r}_2(s) = \langle 2s, 1 + s, 3s \rangle \quad .$$

Setting $\mathbf{r}_1(t) = \mathbf{r}_2(s)$, by equating respective components we have **three** equations and **two** unkowns (s and t):

$$1 + t = 2s \quad , \quad 2t = 1 + s \quad , \quad 3t = 3s \quad .$$

From the third equation we get $t = s$, plugging into the first gives $1 + t = 2t$ so $1 = t$, and so $s = 1$, and plugging into the second equation we get $2 = 2$ which is correct, so we found a solution and this means that these two lines **meet**.

To find **where** they meet you plug-in $t = 1$ into

$$\mathbf{r}_1(t) = \langle 1 + t, 2t, 3t \rangle$$

getting

$$\mathbf{r}_1(1) = \langle 1 + 1, 2 \cdot 1, 3 \cdot 1 \rangle = \langle 2, 2, 3 \rangle \quad .$$

To be on the safe side you should plug-in $s = 1$ into

$$\mathbf{r}_2(s) = \langle 2s, 1 + s, 3s \rangle \quad ,$$

getting

$$\mathbf{r}_2(1) = \langle 2, 3, 3 \rangle \quad ,$$

which is indeed the same. We are almost done!

But beware! They do not meet at $\langle 2, 3, 3 \rangle$, this is nonsense, since they must meet at a point. The **final** step is to convert the vector $\langle 2, 3, 3 \rangle$ into the point $(2, 3, 3)$ (which is the tip of the arrow of the vector (if it starts at the origin)).

Ans.: The two lines do intersect each other, and the intersection point is $(2, 3, 3)$.

Comments: About %70 of the people got it perfectly. A few people did the mathematics perfectly but at the end made the conceptual mistake by saying that they meet at the "point" $\langle 2, 3, 3 \rangle$ rather than $(2, 3, 3)$. Other people were on the right track but messed up the algebra, and quite a few said that they don't meet. Some people said that they meet when $t = 1$. This is correct but not the final answer. Once you get $t = 1$ you have to plug-in into $\mathbf{r}_1(t)$ like we did above.