Solutions to the "QUIZ" for Sept. 28, 2009

1. Find the directional derivative of the function $f(x, y, z) = xy^2z^3$ at the point $(2, 1, 1)$ in the direction $\langle 2, -1, -1\rangle$.

Solution: We first find the **gradient** $\bigtriangledown f$.

$$\bigtriangledown f = \langle\, f_x \,,\, f_y \,,\, f_z \,\rangle \quad .$$

Since $f_x = y^2z^3$, $f_y = 2xyz^3$, $f_z = 3xy^2z^2$, we have

$$\bigtriangledown f = \langle\, y^2z^3 \,,\, 2xyz^3 \,,\, 3xy^2z^2 \,\rangle \quad .$$

Next we plug-in the point $(2, 1, 1)$, i.e. plug-in $x = 2, y = 1, z = 1$.

$$\bigtriangledown f = \langle\, 1^2 \cdot 1^3 \,,\, 2 \cdot 2 \cdot 1 \cdot 1^3 \,,\, 3 \cdot 2 \cdot 1^2 \cdot 1^2 \,\rangle = \langle\, 1 \,,\, 4 \,,\, 6 \,\rangle \quad .$$

Next we have to find the **unit-vector** in the given direction $\langle 2, -1, -1\rangle$. Take the magnitude:

$$||\langle 2, -1, -1\rangle|| = \sqrt{2^2 + (-1)^2 + (-1)^2} = \sqrt{6} \quad .$$

To get the unit vector **u** you divide by the magnitude:

$$\frac{\langle 2, -1, -1\rangle}{\sqrt{6}}$$

Finally, the **directional derivative** is $\bigtriangledown f \cdot u$, giving

$$\langle\, 1 \,,\, 4 \,,\, 6 \,\rangle \cdot \frac{\langle 2, -1, -1\rangle}{\sqrt{6}} = \frac{1 \cdot 2 + 4 \cdot (-1) + 6 \cdot (-1)}{\sqrt{6}} = \frac{-8}{\sqrt{6}} = -\frac{8\sqrt{6}}{6} = -\frac{4\sqrt{6}}{3} \quad .$$

Ans.: $-\frac{4\sqrt{6}}{3}$.

2. Find the maximum rate of change of $f(x, y) = x^2 + y^3$ at the point $(2, 1)$ and the direction in which it occurs.

Sol. We first find the gradient $\bigtriangledown f = \langle\, f_x \,, f_y \,\rangle$. Since $f_x = 2x$, $f_y = 3y^2$, we have

$$\bigtriangledown f = \langle\, 2x \,,\, 3y^2 \,\rangle \quad .$$

Now plug-in $x = 2, y = 1$ getting:

$$\bigtriangledown f = \langle\, 2 \cdot 2 \,,\, 3 \cdot 1^2 \,\rangle = \langle\, 4 \,,\, 3 \,\rangle \quad .$$

The maximum rate of change is the **magnitude**

$$||\bigtriangledown f|| = ||\langle\, 4 \,,\, 3 \,\rangle|| = \sqrt{4^2 + 3^2} = \sqrt{25} = 5 \quad .$$

The direction in which it occurs is the **unit vector** along the direction of the gradient:

$$\frac{\bigtriangledown f}{||\bigtriangledown f||} = \frac{\langle\, 4 \,,\, 3 \,\rangle}{5} = \langle\, \frac{4}{5} \,,\, \frac{3}{5} \rangle.$$

Ans.: The maximum rate of change is 5 and it occurs in the direction $\langle\, \frac{4}{5} \,,\, \frac{3}{5} \,\rangle$.

Note: If you wrote that it occurs in the direction $\langle\, 4 \,,\, 3 \,\rangle$ that's correct too, but it is nice to give the **unit direction**.