Solutions to the "QUIZ" for Sept. 24, 2009

1. Compute the partial derivatives with respect to x and y.

$$z = \ln(x^2 + y^3) \quad .$$

Sol. $\frac{\partial z}{\partial x}$, alias z_x is obtained by treating x as the variable, and y as **constant**. By the chain-rule:

$$z_x = \frac{1}{x^2 + y^3} \cdot (x^2 + y^3)' = \frac{1}{x^2 + y^3} \cdot 2x = \frac{2x}{x^2 + y^3} \quad .$$

Analogously, $\frac{\partial z}{\partial y}$, alias z_y is obtained by treating y as the variable, and x as **constant**. By the chain-rule:

$$z_x = \frac{1}{x^2 + y^3} \cdot (x^2 + y^3)' = \frac{1}{x^2 + y^3} \cdot 3y^2 = \frac{3y^2}{x^2 + y^3} \quad .$$

Comments: About %90 of the people got it right. Some people need to review the chain-rule for functions of a single variable.

2. Find an equation of the tangent plane to the given surface at the specified point.

$$z = x^2 + y^2 + 2 \quad , \quad (1, 1, 4) \quad .$$

Sol.: The formula is:

$$(z - z_0) = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0) \quad .$$

Here $x_0 = 1, y_0 = 1, z_0 = 4$. To make sure that z_0 is correct we do $f(1, 1) = 1^2 + 1^2 + 2 = 4$, that agrees. Now

$$f_x = 2x + 0 + 0 = 2x$$

$$f_y = 0 + 2y + 0 = 2y$$

Plugging-in $x_0 = 1, y_0 = 1$, we get

$$f_x(1, 1) = 2 \cdot 1 = 2 \quad ,$$

$$f_y(1, 1) = 2 \cdot 1 = 2 \quad .$$

Putting it all together, we have

$$(z - 4) = 2(x - 1) + 2(y - 1) \quad .$$

This is already a **correct** answer, but you are supposed to simplify to get:

$$z - 4 = 2x - 2 + 2y - 2 \quad .$$

yielding

$$z = 2x + 2y \quad .$$

Ans.: An equation for the tanglent plane to $z = x^2 + y^2 + 2$ at the point $(1, 1, 4)$ is $z = 2x + 2y$.

Comment:About %95 people got it right. Many people left it as $(z - 4) = 2(x - 1) + 2(y - 1)$. A few people messed-up the addition.