Solutions to the "QUIZ" for Sept. 10, 2009

1. Find an equation of the plane that passes through the points $(0, 1, 1)$, $(1, 0, 1)$, $(1, 1, 0)$.

Sol. Let's call $P = (0, 1, 1)$, $Q = (1, 0, 1)$, $R = (1, 1, 0)$.

We need **two** direction vectors that lie on that plane. For example

$$\mathbf{PQ} = \langle 1 - 0, 0 - 1, 1 - 1 \rangle = \langle 1, -1, 0 \rangle$$

$$\mathbf{PR} = \langle 1, 0, -1 \rangle$$

To get the **normal** we take the **cross-product** $\mathbf{PQ} \times \mathbf{PR}$.

$$\mathbf{PQ} \times \mathbf{PR} =$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} \quad =$$

$$\mathbf{i} \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 1 & 0 \\ 1 & -1 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix}$$

$$= \mathbf{i}((-1) \cdot (-1) - 0 \cdot 0) - \mathbf{j}(1 \cdot (-1) - 0 \cdot 1) + \mathbf{k}(1 \cdot 0 - (-1) \cdot 1) =$$

$$\mathbf{i} + \mathbf{j} + \mathbf{k} \quad .$$

Finally converting to the **usual** notation, we get $\mathbf{n} = \langle 1, 1, 1 \rangle$. The equation of a general plane is

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0 \quad ,$$

where $\langle a, b, c \rangle = \langle 1, 1, 1 \rangle$ is the normal vector we just found, and (x_0, y_0, z_0) is *any* point (either P, Q or R). Picking P we get

$$1(x - 0) + 1(y - 1) + 1(z - 1) = 0 \quad ,$$

that simplies to

$$x + y + z = 2 \quad .$$

Ans.: $x + y + z = 2$.

Comments: About %70 got it pefectly. About %10 were clueless. The rest made some calculational mistakes. Quite a few people got $\mathbf{n} = \langle 1, 1, 1 \rangle$ correctly, but messed up with the last part of plugging-in $(x_0, y_0, z_0) = (0, 1, 1,)$, and got either $x + y + z = 0$, or $x + y + z = 3$ or other wrong things. Remember that you can always **check** your answer by plugging-in the three points and see that they lie on the planne. For $P = (0, 1, 1)$: $0 + 1 + 1 = 2$, for $Q = (1, 0, 1)$, $1 + 0 + 1 = 2$, for $R = (1, 1, 0)$: $1 + 1 + 0 = 2$, they all agree. People who got, for example $x + y + z = 3$ could have realized their mistake by doing this checking.

2. Find the intersection of the line

$$\mathbf{r}(t) = \langle 1, 1, 0 \rangle + t\langle 0, 2, 4 \rangle$$

and the plane

$$x + y + z = 14 \quad .$$

Solution: First **spell-out** $\mathbf{r}(t)$:

$$\mathbf{r}(t) = \langle 1, 1, 0 \rangle + t\langle 0, 2, 4 \rangle = \langle 1, 1 + 2t, 4t \rangle \quad ,$$

and in **scalar** form

$$x = 1 \quad , \quad y = 1 + 2t \quad , \quad z = 4t \quad .$$

Now plug these expressions for x, y, z in terms of the parameter t into the equation of the plane $x + y + z = 14$ getting

$$1 + (1 + 2t) + 4t = 14 \quad .$$

Simplifying, we get

$$2 + 6t = 14 \quad ,$$

so

$$6t = 12 \quad ,$$

that gives $t = 2$. Having found the lucky t (namely 2) you plug it in back into

$$x = 1 \quad , \quad y = 1 + 2t \quad , \quad z = 4t \quad ,$$

getting

$$x = 1 \quad , \quad y = 1 + 2 \cdot 2 = 5, \quad z = 4 \cdot 2 = 8 \quad ,$$

So the **lucky point** that belongs both to the plane and the line is the point $(1, 5, 8)$.

Ans.: The intersection of the line and the plane given by the problem is the point $(1, 5, 8)$.

Comments: About %85 got it perfectly. A few people forgot to plug-in $t = 2$ back into x, y, z. Other people messed up the very simple algebra. These people should review their algebra! BTW, this was problem 32 from section 12.5 that I did it in class (by accident, I picked a random problem and forgot that this was the problem that I picked for the "quiz").