Solutions to the "QUIZ" for Oct. 29, 2009

1. Compute the Jacobian of the transfomation

$$\Phi(r,s) = (rs, r+s)$$

Sol.: Here $x = rs, y = r + s$ and

$$J = (x_r)(y_s) - (x_s)(y_s) = (s)(1) - (r)(1) = s - r \quad .$$

Ans. $s - r$ (type: function of r and s).

Comment: Almost everyone got it right.

2. Let $\mathcal{D} = \Phi(\mathcal{R})$ where $\Phi(u,v) = (u+v, v^2)$ and $R = [0,6] \times [1,2]$. Calculate

$$\int\int_{\mathcal{D}} y \, dA \quad .$$

(Note: it is not necessary to compute D).

Sol. Here the transformation is $x = u + v, y = v^2$. The Jacobian is $J = (x_u)(y_v) - (x_v)(y_u) = (1)(2v) - (1)(0) = 2v$.

By the change of variable formula we have

$$\int\int_{\mathcal{D}} y \, dA = \int\int_{\mathcal{R}} yJ \, dA = \int\int_{\mathcal{R}} (v^2)(2v) \, dA = \int\int_{\mathcal{R}} 2v^3 \, dA$$

R is the rectangle $[0,6] \times [1,2]$, which means:

$$\{(u,v) \,|\, 0 \le u \le 6 \,,\, 1 \le v \le 2\}$$

That is both type I and type II. Using the type-I formulation we have

$$\int_0^6 \int_1^2 2v^3 \, dv \, du = \left(\int_0^6 du \right) \left(\int_1^2 2v^3 \, dv \right) =$$

$$\left(u \Big|_0^6 \right) \left(\frac{v^4}{2} \Big|_1^2 \right) = (6-0) \cdot (\frac{2^4 - 1^4}{2}) = 6 \cdot (\frac{15}{2}) = 45 \quad .$$

Ans. 45 (type number).

Comment: About %60 got it perfectly. Almost everyone set-it up correctly, but a few people got mixed up with the Jacobian, and for some reason got $2v - 1$. Other people, as usual, forgot their multiplication table facts. Some people got confused and took $y = y + v$ rather than $y = v^2$. Don't confuse the x and the y!