

Solutions to the “QUIZ” of Oct. 26, 2009

1. Use polar coordinates to compute the double integral

$$\int \int_D xy \, dA \quad ,$$

where

$$D = \{(x, y) \mid x^2 + y^2 \leq 1, x \geq 0, y \geq 0\} \quad .$$

Sol.: The **region** is the quarter-circle, center origin, radius 1, in the first quadrant. Its **polar coordinates** description is:

$$\{(r, \theta) \mid 0 \leq \theta \leq \pi/2 \quad , \quad 0 \leq r \leq 1\}$$

doing the **translation** we get

$$\int_0^{\pi/2} \int_0^1 (r \cos \theta)(r \sin \theta) r \, dr \, d\theta$$

This is the **most important** part. From here, Maple can do it. Doing it by hand, we have

$$\begin{aligned} \int_0^{\pi/2} \int_0^1 r^3 \cos \theta \sin \theta \, dr \, d\theta &= \left(\int_0^{\pi/2} \cos \theta \sin \theta \, d\theta \right) \left(\int_0^1 r^3 \, dr \right) \\ &= \left(\int_0^{\pi/2} \frac{\sin 2\theta}{2} \, d\theta \right) \left(\int_0^1 r^3 \, dr \right) = \left(\frac{-\cos 2\theta}{4} \Big|_0^{\pi/2} \right) \left(\frac{r^4}{4} \Big|_0^1 \right) \\ &= \frac{-\cos \pi - (-\cos 0)}{4} \cdot \frac{1}{4} = \frac{2}{4} \cdot \frac{1}{4} = \frac{1}{8} \quad . \end{aligned}$$

Ans.: $\frac{1}{8}$ (type=number).

Comment: About %60 of the students were able to set it up correctly. Only about %20 finished it up completely. Many people just left it unevaluated (as I told you to do if you were in a rush).

2. Evaluate the iterated integral by converting it to polar coordinates

$$\int_0^1 \int_0^{\sqrt{1-y^2}} e^{x^2+y^2} \, dx \, dy \quad .$$

Sol. This is an iterated integral over a **type-II** region (since the dy integration is outside). Looking at the limits of integration, the region in question is

$$\{(x, y) \mid 0 \leq y \leq 1, 0 \leq x \leq \sqrt{1-y^2}\} \quad .$$

This is the quarter-circle, center origin, radius 1, that is located at the first-quadrant. A polar-coordinates description is

$$\{(r, \theta) \mid 0 \leq \theta \leq \pi/2, 0 \leq r \leq 1\} \quad .$$

Since $x^2 + y^2 = r^2$ and $dydx = dA = r dr d\theta$, the integral equals:

$$\int_0^{\pi/2} \int_0^1 e^{r^2} r dr d\theta \quad .$$

This is the set-up. In real life you would go to Maple. But here we have to do it by hand.

The inner integral is:

$$\int_0^1 r e^{r^2} dr \quad .$$

Doing the substitution $u = r^2$ we get $du = 2r dr$, and when $r = 0, u = 0$ and when $r = 1, u = 1$. So this equals

$$\int_0^1 e^{u/2} = (1/2)e^u \Big|_0^1 = (1/2)(e^1 - e^0) = \frac{e-1}{2} \quad .$$

Now we do the **outer integral**.

$$\int_0^{\pi/2} \frac{e-1}{2} d\theta = \frac{e-1}{2} \theta \Big|_0^{\pi/2} = \frac{e-1}{2} \cdot \frac{\pi}{2} = \frac{(e-1)\pi}{4} \quad .$$

Ans. $\frac{(e-1)\pi}{4}$ (type number).

Comment: About %50 of the students were able to correctly set it up. About %20 finished it up completely. Many people just left it unevaluated (as I told you to do if you were in a rush).