

Solutions to the “QUIZ” for Oct. 19, 2009

1. Change the order of integration in

$$\int_1^4 \int_0^{\ln y} f(x, y) dx dy \quad .$$

Sol.: The **region** of integration is

$$D = \{(x, y) | 1 \leq y \leq 4, 0 \leq x \leq \ln y\}$$

This is a type II (Horizontally Simple) region. The **main road** is along the y -axis from $(0, 1)$ to $(0, 4)$, and the “side-streets” extend from the y -axis (where $x = 0$) to the curve $x = \ln y$. Drawing it, we get that the projection on the x -axis of that region is between $x = 0$ and $x = \ln 4$, and making this the main road, the vertical side-streets extend from the curve $x = \ln y$, that we should rewrite, from the new perspective as $y = e^x$, to the horizontal line $y = 4$. So a type I description is

$$D = \{(x, y) | 0 \leq x \leq \ln 4, e^x \leq y \leq 4\}$$

Finally the integral is

$$\int_0^{\ln 4} \int_{e^x}^4 f(x, y) dx dy \quad .$$

This is the **ans.** (type: double integral of a general function on a specific domain).

Comments: About 60% of the people got it perfectly. Quite a few people drew a nice diagram, but picked as the region the complement of the true region. Make sure you **highlight** the region. Some people got the type I description, but didn’t set-up the double-integral.

2. Evaluate

$$\int_0^2 \int_{y/2}^1 \frac{1}{(x^2 + 1)^2} dx dy \quad ,$$

by inverting the order of integration and evaluating the new iterated integral.

Sol. The region implied by the given iterated integral is

$$D = \{(x, y) | 0 \leq y \leq 2, y/2 \leq x \leq 1\}$$

This is a type II description. If you draw it, the **main road** is along the y -axis from $y = 0$ to $y = 2$ and the horizontal side streets are from the line $x = y/2$ to the vertical line $x = 1$. This turns out to be a triangle with vertices $(0, 0)$, $(1, 0)$, $(1, 2)$.

Let’s find a type I description of D . The projection on the x -axis is from $x = 0$ to $x = 1$ and the horizontal side streets starts at $y = 0$ and end at the line $x = y/2$ that we now must write as $y = 2x$. So the type I description of D is:

$$D = \{(x, y) | 0 \leq x \leq 1, 0 \leq y \leq 2x\} \quad .$$

Now we can **interchange** the order of integration and get

$$\int_0^1 \int_0^{2x} \frac{1}{(x^2 + 1)^2} dy dx \quad .$$

We first do the **inner** integral

$$\begin{aligned} \int_0^{2x} \frac{1}{(x^2 + 1)^2} dy &= \frac{1}{(x^2 + 1)^2} \left(\int_0^{2x} dy \right) \\ &= \frac{1}{(x^2 + 1)^2} \left(y \Big|_0^{2x} \right) = \frac{1}{(x^2 + 1)^2} (2x - 0) = \frac{2x}{(x^2 + 1)^2} \quad . \end{aligned}$$

Now we are ready to do the **outer** integral

$$\int_0^1 \frac{2x}{(x^2 + 1)^2} dx \quad .$$

This calls for a **u-substitution**. $u = x^2 + 1$, $2x dx = du$, and when $x = 0$, $u = 1$, and when $x = 1$, $u = 2$.

$$\begin{aligned} \int_1^2 \frac{1}{u^2} du &= \int_1^2 u^{-2} du = \frac{u^{-1}}{-1} \Big|_1^2 \\ &= \frac{-1}{u} \Big|_1^2 = \frac{-1}{2} - \frac{-1}{1} = \frac{1}{2} \quad . \end{aligned}$$

Ans.: $\frac{1}{2}$ (type: number).

Comments: About %60 of the people got it perfectly. Some people ran out of time (it was a long quiz).