Solutions to the "QUIZ" for Oct. 15, 2009

1. Calculate the iterated integral

$$\int_1^2 \int_{-1}^1 (x+y^2)\,dx\,dy \quad .$$

Sol. We first do the **inner integral**

$$\int_{-1}^1 (x+y^2)\,dx = \frac{x^2}{2} + y^2 x \Big|_{-1}^1 = (\frac{1^2}{2} + y^2 \cdot 1) - (\frac{(-1)^2}{2} + y^2 \cdot (-1)) = 2y^2$$

Now we do the **outer integral**

$$\int_1^2 \left[\int_{-1}^1 (x+y^2)\,dx\right]\,dy = \int_1^2 2y^2\,dy = \frac{2y^3}{3}\Big|_1^2 = \frac{2\cdot 2^3}{3} - \frac{2\cdot 1^3}{3} = \frac{16}{3} - \frac{2}{3} = \frac{16}{3} - \frac{2}{3} = \frac{14}{3} \quad .$$

Ans.: $\frac{14}{3}$.

Comment: About %90 got it perfectly. As usual, a few people messed up the (easy!) arithmetic, and a couple of people tried "shortcuts", but messed up in using them.

2. Calculate the double integral

$$\int\int_R \frac{x^2 y}{x^3+1}\,dA \quad ,$$

$$R = \{(x,y) \Big| \, 0 \le x \le 1 \, , \, -1 \le y \le 1 \, \} \quad .$$

Sol.: Making it into an iterated integral we have

$$\int_0^1 \int_{-1}^1 \frac{x^2 y}{x^3+1}\,dy\,dx$$

This integrand has the property that it is **separable** i.e. a product of a function of x-alone (namely $x^2/(x^3+1)$) and a function of y-alone (namely y), so it is **legitimate** to use the shortcut:

$$\left(\int_0^1 \frac{x^2}{x^3+1}\,dx\right)\left(\int_{-1}^1 y\,dy\right) \quad .$$

Since the second integral is obviously 0, we don't even have to bother to compute the first integral, since **everything** times zero, is 0 (0 kills everything, and if I know that you are going to die, why bother getting to know you). The answer is 0.

Comment: About %80 got it perfectly. A few people made it harder than it should be, and messed up.