Solutions to the "QUIZ" for Nov. 5, 2009

1. Let C be the line segment from $(0,1)$ to $(2,3)$, find $\int_C xy\,ds$.

Sol.: We first find a *paramteric representation* for the line segment joining $(0,1)$ and $(2,3)$:

$$\mathbf{r}(t) = (1-t)\langle 0,1\rangle + t\langle 2,3\rangle = \langle 0, 1-t\rangle + \langle 2t, 3t\rangle = \langle 2t, 1+2t\rangle$$

Of course **always** (for line segments): $0 \le t \le 1$. So $x = 2t, y = 1+2t$.

Next we find $\mathbf{r}'(t) : \langle 2,2\rangle$, so $|\mathbf{r}'(t)| = \sqrt{2^2+2^2} = \sqrt{8}$, and so $ds = \sqrt{8}dt$. Not we plug in $x = 2t$, $y = 1+2t$ into the integrand, to get:

$$\int_0^1 (2t)(1+2t)\sqrt{8}\,dt = \sqrt{8}\int_0^1 (2t)(1+2t)\,dt =$$

$$\sqrt{8}\int_0^1 2t + 4t^2\,dt = \sqrt{8}(t^2 + 4\frac{t^3}{3})\Big|_0^1 = \sqrt{8}(1^2 + 4\frac{1^3}{3}) - 0 = \sqrt{8}\frac{7}{3} = \sqrt{2}\frac{14}{3} \quad .$$

Ans.: $\sqrt{2}\frac{14}{3}$.

Comments: About %30 got it completely. Another %40 followed the right way, but messed up the calculations. Some people had trouble with the first part of finding the paratmetric representation of the line segment.

2. Evaluate

$$\int_C xy^2\,dx + x^2y\,dy \quad ,$$

where C is $x : t^2$, $y = t^3$, $0 \le t \le 1$.

Sol. $dx = (t^2)'\,dt = 2t\,dt$, $dy = (t^3)'\,dt = 3t^2\,dt$. So we have

$$\int_0^1 (t^2)(t^3)^2(2t)\,dt + (t^2)^2(t^3)(3t^2)\,dt = \int_0^1 2t^9\,dt + 3t^9\,dt = \int_0^1 5t^9\,dt = \frac{t^{10}}{2}\Big|_0^1 = \frac{1}{2} \quad .$$

Ans.: $\frac{1}{2}$.

Comment: About %85 of the students got it completely.