Solutions to the "QUIZ" for Nov. 2, 2009

1. Sketch the planar vector field

$$\mathbf{F} = \langle x, y^2 \rangle \quad .$$

Solution: The way that you are supposed to do it is to pick a few sample points.

At the point $(0,0)$, the vector is $\langle 0, 0^2 \rangle = \langle 0, 0 \rangle$, that is the zero vector, so you only draw a dot.

At the point $(1,0)$ the vector is $\langle 1, 0^2 \rangle = \langle 1, 0 \rangle$, so you draw an arrow starting at the point $(1,0)$ and ending at the point $(2,0)$.

At the point $(1,1)$ the vector is $\langle 1, 1^2 \rangle = \langle 1, 1 \rangle$, so you draw an arrow from the point $(1,1)$ to the point $(1+1, 1+1) = (2,2)$.

At the point $(2,3)$ the vector is $\langle 2, 3^2 \rangle = \langle 2, 9 \rangle$, so you draw an arrow from the point $(2,3)$ to the point $(2+2, 3+9) = (4,12)$. Etc.

Comments: Only about %50 of the students knew what to do, but that was because I covered it too fast. I will review this again on Thursday.

2. Find a potential function for the vector field $\mathbf{F}$

$$\mathbf{F} = \langle \, y\cos(xy) \, , \, x\cos(xy) \, \rangle \quad .$$

Solution. Next time we will learn how to to do it **systematically**. Stricly speaking, we had to first check that $\mathbf{F}$ is a **gradient vector field**, by checking that

$$\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x} \quad .$$

You are welcome to do it, but **this** question didn't ask for that step.

Next time we will learn a **method** for finding the potential function $\phi(x,y)$ (whenever it exists), but in this section you are supposed to do it "by inspection", and than verify that your guess is correct, by checking that

$$\mathbf{F} = \nabla\phi \quad .$$

The condition is that

$$\frac{\partial \phi}{\partial x} = F_1 \quad , \quad \frac{\partial \phi}{\partial y} = F_2 \quad .$$

In other words

$$\frac{\partial \phi}{\partial x} = y\cos(xy) \quad , \quad \frac{\partial \phi}{\partial y} = x\cos(xy) \quad .$$

Since the **antiderivative** of **cosine** is **sine**, a reasonable guess is that

$$\phi(x, y) = \sin(xy) \quad .$$

Let's confirm our guess.

$$\phi_x = \cos(xy) \cdot y = y \cos(xy) \quad ,$$

$$\phi_y = \cos(xy) \cdot x = x \cos(xy) \quad .$$

So it agrees.

Ans. The potential function is $\phi(x, y) = \sin(xy)$.

Comments. Only about %30 of the students got it correctly. It is a tricky question, and next time we will learn how to do it systematically.