

Solutions to the ‘QUIZ’ for Nov. 16, 2009

1. Compute the line-integral of the scalar function $f(x, y, z) = z^2$ over the curve $\mathbf{c}(t) = (2t, 3t, 4t)$ for $0 \leq t \leq 2$.

Sol.

$$\mathbf{c}'(t) = \langle 2, 3, 4 \rangle$$

So

$$|\mathbf{c}'(t)| = |\langle 2, 3, 4 \rangle| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$$

So

$$\begin{aligned} \int_C z^2 ds &= \int_0^2 (4t)^2 \sqrt{29} dt = \int_0^2 16t^2 \sqrt{29} dt = \\ &= 16\sqrt{29} \frac{t^3}{3} \Big|_0^2 = 16\sqrt{29} \frac{2^3}{3} = \frac{128\sqrt{29}}{3} . \end{aligned}$$

Ans.: $\frac{128\sqrt{29}}{3}$.

Comment: About %90 of the people got it right.

2. Compute the Jacobian of

$$\Phi(u, v) = (3u + 4v, u - 2v) .$$

Sol. Here $x = 3u + 4v, y = u - 2v$.

$$J = (x_u)(y_v) - (x_v)(y_u) = (3)(-2) - (4)(1) = -6 - 4 = -10 .$$

Ans. -10 . (Type: function of (u, v) that happens to be a constant function.)

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