Solutions to the "QUIZ" for Dec. 7, 2009

1. Let

$$F(x,y,z) =$$
$$\langle \cos(\sqrt{1+x^7}+zy^9) \quad , \quad \tan(x^7+y^2+1/z) \quad , \quad \tan^{-1}(e^{xyz}+\cos^6(x^8-y+3z)\,\rangle \quad ,$$

and let $\langle P, Q, R \rangle = curl\ \mathbf{F}$. Compute

$$\frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y}+\frac{\partial R}{\partial z} \quad .$$

Be sure to explain everything. (Note: there was a typo in the problem that has now been corrected).

Sol. We have to compute $div\,(\langle P,Q,R\rangle)$ but $\langle P,Q,R\rangle = curl(\mathbf{F})$ so we have to compute $div(curl(\mathbf{F}))$. But a famous theorem says that this is always the *zero* function.

Ans.: 0.

Comment: Almost everyone got it right.

2. Calculate the surface integral
$\int\int_S \mathbf{F}\cdot d\,\mathbf{S}$, where

$$\mathbf{F}(x,y,z) = \langle\, 2x+y+z \,,\, x+2y+z \,,\, x+y+2z \,\rangle$$

where S is the surface of the box bounded by the planes $x=0$, $x=1$, $y=0$, $y=4$, $z=0$, $z=5$.

Sol.: We are supposed to use the **Divergence Theorem**:

$$\int\int_S \mathbf{F}\cdot d\,\mathbf{S} \;=\; \int\int\int_E div(\mathbf{F})dV \quad ,$$

where E is the **inside** of the box. We have

$$div(\mathbf{F}) = \frac{\partial(2x+y+z)}{\partial x}+\frac{\partial(x+2y+z)}{\partial y}+\frac{\partial(x+y+2z)}{\partial z} = 2+2+2=6 \quad .$$

So

$$\int\int_S \mathbf{F}\cdot d\,\mathbf{S} \;=\; \int\int\int_E 6dV \quad ,$$

Taking 6 out, we get:

$$6\int\int\int_E 1dV \quad .$$

But a **volume-integral** whose integrand is 1 equals the volume, so this equals

$$6\cdot Volume(Box)$$

The volume of the box is $1\cdot 4\cdot 5 = 20$, so the answer is $6\cdot 20 = 120$.

Ans.: 120.

Comment: About %95 of the students got it right.