NAME: (print!) ____

Section: ____ **E-Mail address:** ____________________

MATH 251 (1-6,10,11), Dr. Z., Fall 2009, First Practice Test for Exam 2

Do not write below this line (office use only)

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

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total (out of 100)

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1. (10 pts.) (i) Is the following

$$\mathbf{F} = \langle\, 2\cos(2x+3y+5z) + y^3 + 1\,,\; 3\cos(2x+3y+5z) + 3xy^2 + 1\,,\; 5\cos(2x+3y+5z)\,\rangle$$

a conservative vector field?

(ii) If it is, find a function $f(x, y, z)$ such that $\mathbf{F} = \bigtriangledown f$.

(iii) Compute $3f(1, 1, -1) - 3f(0, 0, 0) - 2$

Ans. to (iii):

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2. (10 points) Show that the line integral

$$\int_C 2x \sin y \, dx \; + \; (x^2 \cos y - 3y^2) \, dy \quad ,$$

is independent of the path C, and evaluate it if C is *any* path from $(1, 0)$ to $(0, 2)$.

Ans.:

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3. (10 points) Evaluate

$$\int\int\int_E 10(x^2+y^2)^4 \, dV \quad ,$$

where E is the solid that lies within the cylinder $x^2+y^2=4$, below the plane $z=0$, and above the cone $z^2=x^2+y^2$.

Ans.:

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4. (10 points)
Use the transformation

$$x = 2u + v \quad , \quad y = u + 2v \quad ,$$

to evaluate the integral

$$\int\int_R (2x - y)\, dA$$

where R is the triangular region with vertices $(0,0)$, $(2,1)$, and $(1,2)$.

Ans.:

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5. (10 points) Use the given transformation to evaluate the integral

$$\int\int_R 4\sqrt{2x+y}\,dA \quad ,$$

where R is the triangular region with vertices $(0,0)$,$(4,-6)$, $(6,-10)$; $x = 3u - v$, $y = -5u + 2v$.

Ans.:

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6. (10 points) Evaluate the iterated integral by converting it to polar coordinates

$$\int_0^3 \int_0^{\sqrt{9-x^2}} e^{x^2+y^2}\, dy\, dx \quad .$$

Ans.:

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7. (10 points) Let A be the **number**

$$A = \int_0^9 \int_{y/3}^3 e^{x^2}\, dx\, dy \quad .$$

What is $2A + 3$? (Hint: Not even Dr. Z. can do $\int e^{x^2}\, dx$, so you must be clever, and first change the order of integration.)

Ans.:

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8. (10 points) Find the volume of the solid bounded by the cylinder $y = x^4$ and the planes $z = 0$ and $y + z = 1$. Simplify as much as you can.

Ans.:

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9. (10 points) Use Lagrange multipliers (no credit for other methods) to find the maximum value of the function $f(x, y) = x^2y^3$ subject to the constraint $x + y = 10$.

Ans.:

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10. (10 points) Use Largange multipliers (no credit for other methods) to find the largest value that $x + 3y + 5z$ can be, given that $x^2 + y^2 + z^2 = 35$

Ans.:
