NAME: (print!) ____

Section: ____ **E-Mail address:** ____________________

MATH 251 (1-6,10-11), Dr. Z. , Third Practice for Exam 1 (Version of Oct. 7, 2009, 4:28pm, thanks to Britanie Peabody)
(Previous version Oct. 6, 2009, 1:20pm, thanks to Jonathan Liu!)

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM
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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

Types: Number, Function of *variable*(s), 2D vector of numbers, 3D vector of numbers, 2D vector of functions, 3D vector of functions, equation of a plane, parametric equation of a line, equation of a line, equation of a surface, equation of a line, DNE (does not exist).

1. (10 points) Use the chain rule to find f_u and f_v if

$$f(x,y) = x^3 + y^3 \quad , \quad x = e^{u+v} \quad , \quad y = 2u + 3v \quad .$$

Express your answer in terms of u and v.

The **types** of the answer are:

2. (10 points) Find $\frac{\partial f}{\partial r}$ and $\frac{\partial f}{\partial s}$

$$f(x, y) = x^3 + xyz + y^3 + z^3 \quad ,$$

and the variables are related by $x = r^2 + 2s$ and $y = 3r + 2s^2$. You can leave your answer in terms of x, y, z as well as r, s, and you do not need to simplify!

The **type** of the answers is:

3. (10 points) Find the maximal rate of change of $f(x, y, z) = x^2y^3z^4$ at $(1, 1, 1)$, and the **unit** direction where it occurs.

The **types** of the answers are:

4. (10 points) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if

$$x^3 + y^3 + e^{x+y+z} = 5\cos(x + y + z) + 1 \quad .$$

The **types** of the answer are:

5. (10 points) Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ if

$$z^3 + x^3 + y^3 + 3xyz = 6 \quad .$$

The **types** of the answer are:

6. (10 points) Find the limit if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{5x+2y}{2x+3y+1} \quad .$$

The **type** of the answer is:

7. (10 points) A certain particle has law of motion

$$\mathbf{r}(t) = \langle \sin t \, , \, \cos 2t \, , \, e^t \, \rangle \quad ,$$

Find its velocity, acceletartin, and speed at $t = \pi/6$.

The **types** of the answers are:

8. (10 points) Find the limit if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x^{10}}{(x^2+y^2)^2} \quad .$$

The **type** of the answer is:

9. (10 points, altogether) Do the following limits exist? If they do, find them. Explain!

The **types** of the answers are: and .

a. (5 points) If

$$\lim_{(x,y,z)\to(1,2,3)} f(x,y,z) = 1 \quad , \quad \lim_{(x,y,z)\to(1,2,3)} g(x,y,z) = 2$$

compute

$$\lim_{(x,y,z)\to(1,2,3)} (f(x,y,z) + g(x,y,z))^3 e^{g(x,y,z)}$$

b. (5 points)

$$\lim_{(x,y)\to(0,0)} \frac{x^4 + y^4}{2x^4 + y^4} \quad .$$

10. (10 pts.) Compute the partial derivatives with respect to x and y.

$$z = xy \ln(x^3 + xy + y^3) \quad .$$

The **types** of the answers are:

Answers: 1. $f_u = 3e^{3(u+v)} + 6(2u+3v)^2$, $f_v = 3e^{3(u+v)} + 9(2u+3v)^2$. (Type: functions of u, v).
2. $f_r = 2(3x^2 + yz)r + 3(xz + 3y^2)$, $f_s = 2(3x^2 + yz) + 4s(xz + 3y^2)$. (Type: functions).
3. $\sqrt{29}$, $\langle \frac{2}{\sqrt{29}} \ \frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}} \rangle$ (types: number, vector of numbers).
4. $\frac{\partial z}{\partial x} = -\frac{3x^2 + e^{x+y+z} + 5\sin(x+y+z)}{e^{x+y+z} + 5\sin(x+y+z)}$, $\frac{\partial z}{\partial y} = -\frac{3y^2 + e^{x+y+z} + 5\sin(x+y+z)}{e^{x+y+z} + 5\sin(x+y+z)}$. (Type: functions of x, y (but also using z, that is an implicitly defined function of (x, y))).
5. $\frac{\partial z}{\partial x} = -\frac{3x^2 + 3yz}{3z^2 + 3xy}$, $\frac{\partial z}{\partial y} = -\frac{3y^2 + 3xz}{3z^2 + 3xy}$. (Type: functions of x, y (but also using z, that is an implicitly defined function of (x, y))).
6. 0 (type: Number).
7. $\mathbf{v} = \langle \frac{\sqrt{3}}{2}, -\sqrt{3}, e^{\pi/6} \rangle$, $\mathbf{a} = \langle -\frac{1}{2}, -2, e^{\pi/6} \rangle$, speed= $\sqrt{\frac{15}{4} + e^{\pi/3}}$ (types: vector of numbers, vector of numbers, positive number).
8. 0 (type: Number). [Use Polar coordinates].
9. (a) $27e^2$ (type: Number). (b): DNE (type: DNE).
10.

$$\frac{\partial z}{\partial x} = y\ln(x^3 + xy + y^3) + \frac{xy(3x^2 + y)}{x^3 + xy + y^3}$$

$$\frac{\partial z}{\partial y} = x\ln(x^3 + xy + y^3) + \frac{xy(x + 3y^2)}{x^3 + xy + y^3}$$