NAME: (print!) _____

Section: ___ **E-Mail address:** _____

MATH 251 (1-6,10-11), Dr. Z. , First Practice For Exam 1 (Version of Oct. 6, 2009, 9:40am (Thanks to Holly Bothelo))

FRAME YOUR FINAL ANSWER(S) TO EACH PROBLEM
Do not write below this line

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1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

Types: Number, Function of *variable*(s), 2D vector of numbers, 3D vector of numbers, 2D vector of functions, 3D vector of functions, equation of a plane, parametric equation of a line, equation of a line, equation of a surface, equation of a line, DNE (does not exist).

1. (10 pts.) Find the curvature of the curve

$$\mathbf{r}(t) = \langle t^3, t^2, t \rangle$$

at the point $(1, 1, 1)$.

The **type** of the answer is:

2. Compute the length of the curve

$$\mathbf{r}(t) = \langle\, 2t\,,\, \ln t\,,\, t^2\,\rangle \quad , 1 \leq t \leq 4$$

3. (10 points) Find the directional derivative of $f(x, y, z) = x^3 + y^3 + z^3 + 3xyz$ at $P = (1, 1, 1)$ in the direction pointing to the point $Q = (1, 2, 3)$.

The **type** of the answer is:

4. (10 points) Find an equation of the tangent plane to the surface

$$z = x^3 + y^3 + 2xy$$

at the point $(1, 1, 4)$.

The **type** of the answer is:

5. (10 points) Use the linear approximation to estimate the value of $\sqrt{3.01^2 + 3.99^2}$.

The **type** of the answer is:

6. (10 points) Compute the higher-order partial derivatives $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial y^2}$, $\frac{\partial^2 z}{\partial x \partial y}$ of $z = e^{-x^2-y^2}$.

The **type** of the answer is:

7. (10 points, altogether) Do the following limits exist? If they do, find them. Explain!

The **types** of the answers are: and .

a. (4 points)

$$\lim_{(x,y,z)\to(1,1,1)} \frac{x+y+z}{xy+xz+yz}$$

b. (6 points)

$$\lim_{(x,y)\to(0,0)} \frac{x^2+5y^2}{3x^2+y^2} \quad .$$

8. (10 points) Evaluate the limit or prove that it does not exist

$$\lim_{(x,y)\to(0,0)} \frac{x^6}{x^2+y^2} \quad .$$

The **type** of the answer is:

9. (10 points) A certain particle has acceleration

$$\mathbf{a}(t) = \langle\, 2\,,\, -\sin t\,,\, 4\cos 2t\,\rangle \quad ,$$

and at $t = 0$ its velocity is $\langle 0, 1, 0\rangle$ and its position vector is $\langle\, 0\,,\, 0\,,\, -1\,\rangle$, find its velocity and position vector at time $t = \pi$. Also find the speed at $t = \pi$.

The **type** of the answer is:

10. (10 points) Find an equation to the plane that contains the line

$$x = 1 + 2t \quad , \quad y = 2 + t \quad , \quad z = -1 + 3t$$

and also contains the origin.

The **type** of the answer is:

Answers 1. $\frac{\sqrt{76}}{14\sqrt{14}}$ (type: Number).

2. $15 + \ln 4$ (type: Number).

3. $\frac{18}{\sqrt{5}}$ (type: Number).

4. $z = 5x + 5y - 6$ (type: Equation of a Plane).

5. $\frac{2499}{500}$ (type: Number).

6. $2(2x^2 - 1)e^{-x^2-y^2}$, $2(2y^2 - 1)e^{-x^2-y^2}$, $4xye^{-x^2-y^2}$ (type: functions).

7. 1 (type: number); Does Not Exist (type: DNE).

8. 0 (type: number).

9. velocity is $\langle 2\pi, -1, 0\rangle$, position is $\langle \pi^2, 0, -1\rangle$ (both types are "vectors of numbers"). Speed is $\sqrt{1 + 4\pi^2}$ (type: Number).

10. $7x - 5y - 3z = 0$ (type: equation of plane).