

NAME: \_\_\_\_\_ Section: \_\_\_\_\_

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**MATH 251 (4-6,11), Dr. Z. , Exam 2, 12:00noon-1:20pm , Thurs. Nov. 19, 2009, SEC 117**

**No Calculators!**

Do not write below this line (office use only)

\_\_\_\_\_ 1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

\_\_\_\_\_

**total:** (out of 100)

1. By finding a function  $f$  such that  $\mathbf{F} = \nabla f$ , evaluate  $\int_C \mathbf{F} \cdot d\mathbf{r}$  along the given curve  $C$ .

$$\mathbf{F}(x, y, z) = y^2 z^3 \mathbf{i} + 2xyz^3 \mathbf{j} + 3xy^2 z^2 \mathbf{k} \quad ,$$

$$C : x = 2t \quad , \quad y = t^2 \quad , \quad z = t^5 \quad , \quad 0 \leq t \leq 1 \quad .$$

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**Ans:**

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**2.** Evaluate the line integral

$$\int_C 5y \, dx + 5x \, dy + 6xyz \, dz \quad ,$$

where  $C : x = t, y = t^2, z = t^3, 0 \leq t \leq 1$ .

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**Ans.:**

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**3.** Evaluate

$$\int \int \int_E \frac{5}{62\pi} (x^2 + y^2 + z^2) dV \quad ,$$

where  $E$  is bounded by the  $yz$ -plane and the hemispheres  $x = \sqrt{1 - y^2 - z^2}$  and  $x = \sqrt{4 - y^2 - z^2}$ .

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**Ans.:**

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4. Evaluate the triple integral

$$\int \int \int_E \frac{160}{3 \sin 1} yz \cos(x^5) dV \quad ,$$

where

$$E = \{(x, y, z) \mid 0 \leq x \leq 1, 0 \leq y \leq x, x \leq z \leq 2x\} \quad .$$

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**Ans.:**

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5. Find the Jacobian of the transformation from  $(u, v, w)$ -space to  $(x, y, z)$ -space.

$$x = -uv^2 \quad , \quad y = uw^2 \quad , \quad z = v,$$

at the point  $(u, v, w) = (1, 1, 1)$ .

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**Ans.:** 2

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6. Evaluate the integral

$$\iint_D \frac{16e}{\pi(e-1)} e^{-x^2-y^2} dA \quad ,$$

where  $D$  is the region bounded by the semi-circle  $y = -\sqrt{1-x^2}$  and the  $x$ -axis.

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**Ans.:**

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7. Calculate the iterated integral

$$\int_1^2 \int_0^1 (2x^3 + 2y) \, dx \, dy \quad .$$

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**Ans.:**

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8. Use Lagrange multipliers to find the maximum and minimum values of  $f(x, y) = x + 2y - 2$  subject to the constraint  $x^2 + y^2 = 20$ .

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**maximum value:**

**minimum value:**

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**9.** Find the local maximum and minimum **values** and saddle point(s) of the function  $f(x, y) = (1 + xy)(x + y)$

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**local maximum value(s):**

**local minimum value(s):**

**saddle point(s):**

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**10.** Sketch the region of integration and change the order of integration.

$$\int_0^2 \int_{2x}^4 F(x, y) \, dy \, dx$$

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**Ans.:**

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