NAME: (print!) ____ **Section:** ____

E-Mail address: ____________________

MATH 251 (1-3,10), Dr. Z. , Exam 2, Thurs., Nov. 19, 2009, 10:20-11:40am, SEC 118

No Calculators!

Do not write below this line (office use only)

----

1. (out of 10)

2. (out of 10)

3. (out of 10)

4. (out of 10)

5. (out of 10)

6. (out of 10)

7. (out of 10)

8. (out of 10)

9. (out of 10)

10. (out of 10)

----

total (out of 100)

----

1. (10 pts.) (i) Prove that

$$\mathbf{F} = \langle 2e^{2x+3y+5z} + y^2 + 1 \, , \, 3e^{2x+3y+5z} + 2xy + 1 \, , \, 5e^{2x+3y+5z} \rangle$$

is a conservative vector field.

(ii) Find a function $f(x, y, z)$ such that $\mathbf{F} = \bigtriangledown f$.

(iii) Compute $f(1, 1, 1) - f(0, 0, 0) - e^{10} + 1$

Ans. to (iii):

---

2. (10 points) Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r} \quad ,$$

where C is given by the vector function $\mathbf{r}(t)$.

$$\mathbf{F}(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k} \quad ,$$

$$\mathbf{r}(t) = t^3\mathbf{i} + t^2\mathbf{j} + \mathbf{k} \quad , \quad 0 \le t \le 1 \quad .$$

Ans.:

---

3. (10 points) Evaluate

$$\int\int\int_E \frac{9}{\pi}(x^2+y^2)^3\,dV \quad ,$$

where E is the solid that lies within the cylinder $x^2+y^2=1$, above the plane $z=0$, and below the cone $z^2=4x^2+4y^2$.

Ans.:

---

4. (10 points) Evaluate the iterated integral

$$\int_0^1 \int_x^{2x} \int_0^{x+y} \frac{24z}{19}\, dz\, dy\, dx \quad .$$

Ans.:

---

5. (10 points) Use the given transformation to evaluate the integral

$$\int\int_R 20(2x+y)^2\,dA \quad ,$$

where R is the triangular region with vertices $(0,0)$,$(2,-3)$, $(3,-5)$; $x = 3u - v$, $y = -5u + 2v$.

Ans.:

---

6. (10 points) Evaluate the iterated integral by converting to polar coordinates.

$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \frac{27}{64\pi}(x^2+y^2)^2 \, dy \, dx$$

Ans.:

---

7. (10 points) Let A be the **number**

$$A = \int_0^4 \int_{y/2}^2 e^{x^2} \, dx \, dy \quad .$$

What is $1 + e^4 - A$? (Hint: Not even Dr. Z. can do $\int e^{x^2} \, dx$, so you must be clever, and first change the order of integration.)

Ans.:

---

8. (10 points) Calculate the double integral

$$\int\int_R \frac{27x^2y^2}{(\ln 2)(x^3+1)}\, dA \quad ,$$

$$R = \{(x,y)\,|\, 0 \le x \le 1\,,\, -1 \le y \le 1\,\} \quad .$$

Ans.:

---

9. (10 points) Use Lagrange multipliers to find the maximum value of the function $f(x, y) = x^2y - 27$ subject to the constraint $x + y = 6$.

Ans.:

---

10. (10 points) Find the local maximum and minimum **values**, and saddle point(s) of the function $f(x, y) = x^4 + y^4 - 4xy + 5$.

Local maximum value(s):

Local minimum value(s)

saddle point(s):
