NAME: (print!) ____

Section: ___ **E-Mail address:** ______________________

MATH 251 (4-6,11), Dr. Z. , Final Exam , Mon., Dec. 21, 2009, SEC 111, 12:00-3:00pm

WRITE YOUR FINAL ANSWER TO EACH PROBLEM IN THE INDICATED PLACE (right under the question)

Do not write below this line

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1. (out of 10)
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20. (out of 10)

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tot. (out of 200)

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1. (10 pts.) Find the curvature of the curve

$$\mathbf{r}(t) = \langle e^t, \sin t, \cos t \rangle$$

at the point $(1, 0, 1)$.

Ans.::

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2. Sketch the region of integration and change the order of integration.

$$\int_0^1 \int_0^x F(x,y)\,dy\,dx + \int_1^2 \int_0^1 F(x,y)\,dy\,dx + \int_2^3 \int_0^{3-x} F(x,y)\,dy\,dx$$

Ans.::

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3. (10 points) Find the absolute maximum value of the function $f(x, y) = x^2 + y^2 - 2x - 2y + 2$ in the triangular region

$$\{(x, y) \mid x \geq 0\,,\ y \geq 0\,,\ x + y \leq 4\} \quad .$$

Ans.::

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4. (10 points) Find an equation of the tangent plane to the surface

$$z^5 + x^4 - y^3 = 1 \quad ,$$

at the point $(1, 1, 1)$

Ans.::

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5. (10 points) Compute $f_{xy}(1,1)$ if $f(x,y) = e^{x^2+xy+y^2 y}$.

Ans.::

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6. (10 points) Use the linearization of $f(x, y) = \sqrt{x^2 + y^2 + 1}$ to approximate $f(2.01, 1.98)$.

Ans.::

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7. (10 points) Does the following limit exist? If it does, find it. If it does not, explain why it does not exist.

$$\lim_{(x,y,z)\to(0,0,0)} \frac{x^3+y^3+z^3}{2x^3+3y^3+4z^3}$$

Ans.::

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8. (10 points) Find the length of the curve

$$\mathbf{r}(t) = \langle\, 4t\,,\, 3\sin t\,,\, 3\cos t\,\rangle \quad 0 \le t \le \pi \quad .$$

Ans.::

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9. (10 points) A certain particle has acceleration

$$\mathbf{a}(t) = \langle\, 16e^{4t} \,,\, 9\sin(3t) \,,\, 6t \,\rangle \quad ,$$

and at $t = 0$ its velocity is $\langle 4, -3, 0\rangle$ and its position vector is $\langle 1, 0, 0\rangle$, find its position vector at any time t.

Ans.::

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10. (10 points) Let $\Phi(u, v) = (3u + v, u - 2v)$. Use the Jacobian to determine the area of $\Phi(\mathcal{R})$ for $\mathcal{R} = [2, 5] \times [1, 7]$.

Ans.::

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11. (10 pts.) Compute *curl* $\mathbf{F}$ if

$$\mathbf{F} \,=\, \langle\, 2e^{2x+3y+5z} + y^2 + 2y + 1\,,\; 3e^{2x+3y+5z} + 2xy + 3x + 1\,,\; 5e^{2x+3y+5z}\,\rangle$$

Ans.::

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12. (10 points) Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r} \quad ,$$

where C is given by the vector function $\mathbf{r}(t)$.

$$\mathbf{F}(x, y, z) = y\mathbf{i} + z\mathbf{j} + y\mathbf{k} \quad ,$$

$$\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j} + 2t\mathbf{k} \quad , \quad 0 \leq t \leq 1 \quad .$$

Ans.:

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13. (10 points) Evaluate

$$\int\int\int_E \frac{1}{\sqrt{x^2+y^2}}\, dV \quad ,$$

where E is the solid that lies within the cylinder $x^2+y^2=1$, above the plane $z=0$, and below the cone $z^2=4x^2+4y^2$.

Ans.:

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14. (10 points) Evaluate the iterated integral

$$\int_0^2 \int_x^{3x} \int_0^{x+y} x \, dz \, dy \, dx \quad .$$

Ans.:

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15. (10 points) Use the given transformation to evaluate the integral

$$\int\int_R (2x+y)^2 \, dA \quad ,$$

where R is the triangular region with vertices $(0,0)$,$(2,-3)$, $(3,-5)$; $x = 3u - v$, $y = -5u + 2v$.

Ans.:

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16. (10 points) Evaluate the iterated integral by converting to polar coordinates.

$$\int_0^1 \int_{-\sqrt{1-x^2}}^0 10(x^2+y^2)^4 \, dy \, dx$$

Ans.:

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17. (10 points) Compute the (vector-field) line-integral

$$\int_C \mathbf{F} \cdot d\mathbf{r} \quad ,$$

where C is an ellipse with semi-major-axis 4 and semi-minor-axis 3, travelled **clockwise**, and the vector-field $\mathbf{F}$ is

$$\mathbf{F} = \langle\, 2y \,,\, 3x \,\rangle \quad .$$

(Reminder: the area of an ellipse with semi-major axis a and semi-minor-axis b equals πab).

Ans.:

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18. (10 points) Compute the vector-field surface integral:

$$\int\int_S \mathbf{F} \cdot d\mathbf{S} \quad ,$$

where

$$\mathbf{F} = \langle 5x \, , \, 2y \, , \, 3z \, \rangle$$

and where S is the bounardy of a pyramid whose base is a 2×2 square and whose height is 3, with the normal pointing **outward**.

(Reminder: the volume of a pyramid is the area of the base times the height divided by 3)

Ans.:

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19. (10 points) Find the maximum value of the function $f(x, y) = x^2y^4$ subject to the constraint $x^2 + 2y^2 = 6$.

Ans.:

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20. (10 points) Find the local maximum and minimum **values**, and saddle point(s) of the function $f(x, y) = e^x - xe^y$.

Local maximum value(s):

Local minimum value(s)

saddle point(s):
