NAME: (print!) ____

Section: _______ **E-Mail address:** ________________________

MATH 251 (1-3,10), Dr. Z. , Final Exam , Fri., Dec. 18, 2009, SEC 111, 8:00-11:00am

WRITE YOUR FINAL ANSWER TO EACH PROBLEM IN THE INDICATED PLACE (right under the question)

Do not write below this line

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1. (out of 10)
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tot. (out of 200)

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1. (10 points) Compute the line-integral

$$\int_C 7y\,dx + 3x\,dy \quad ,$$

where C is the closed path that consists from the line segment from $(0,0)$ to $(1,2)$ followed by the line segment from $(1,2)$ to $(2,0)$ followed by the line segment from $(2,0)$ back to $(0,0)$.

Ans.:

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2. (10 points) Find an equation of the tangent plane to the surface

$$\Phi(u, v) = (u^2 + v^2, 2uv, u^3) \quad ,$$

at the point $(5, 4, 1)$.

Ans.:

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3. (10 points) Find the absolute maximum value and the absolute minimum value of the function $f(x,y) = x^2 + y^2$ in the region

$$\{(x,y) \mid -1 \leq x \leq 1, -1 \leq y \leq 1\}.$$

Absolute minimum value:

Absolute maximum value:

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4. (10 points) Compute f_{xy} if

$$f(x, y) = \sin(x^2 + xy + y^2) \quad .$$

Ans.:

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5. (10 points) Find $\frac{\partial z}{\partial x}$ at the point $(1, 1, 1)$ if (x, y, z) are related by:

$$x^3y^3z^3 + 3xyz = 4$$

Ans.:

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6. (10 points) Find an equation for the plane that contains both the line

$$x = 1 + t\,,\ y = 2 + t\,,\ z = 3 + t \quad (-\infty < t < \infty)$$

and the line

$$x = -t\,,\ y = 1 + t\,,\ z = 2 + t \quad (-\infty < t < \infty)$$

Ans.:

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7. (10 points) A certain particle has law of motion

$$\mathbf{r}(t) = \langle\, 2\sin t\,,\ 2\cos 2t\,,\ 2\sin 3t\,\rangle \quad ,$$

Find its acceleration at $t = \pi/6$.

Ans.:

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8. (10 points) Compute the (scalar-function) line-integral

$$\int_C y\, ds$$

where the curve C is given by the parametric equation:

$$\mathbf{r}(t) = \langle\, 1 + \sin 5t\,,\ 1 + \cos 5t\,,\ \sqrt{75}t\,\rangle \quad , \quad 0 \le t \le 1 \quad .$$

Ans.:

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9. (10 points)

If

$$\lim_{(x,y,z)\to(1,2,3)} f(x,y,z) = 1 \quad , \quad \lim_{(x,y,z)\to(1,2,3)} g(x,y,z) = \pi/6$$

compute

$$\lim_{(x,y,z)\to(1,2,3)} \left(f(x,y,z) + \cos(2g(x,y,z))\right)^2 \sin(g(x,y,z))$$

Ans.:

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10. Compute

$$\int\int_S \mathbf{F} \cdot d\mathbf{S} \quad ,$$

where

$$\mathbf{F} = \langle x^2 + \sin(y+z)\,,\, y^2 + xz^3\,,\, z^2 + e^{xy}\,\rangle$$

and where S is the boundary (consisting of all six faces) of the cube

$$\{(x,y,z)\,|\,0 \le x,y,z \le 1\}$$

with the normal pointing **outward**.

Ans.:

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11. By finding a function f such that $\mathbf{F} = \bigtriangledown f$, evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve C.

$$\mathbf{F}(x,y,z) = 2e^{2x+3y+4z}\,\mathbf{i} + 3e^{2x+3y+4z}\mathbf{j} + 4e^{2x+3y+4z}\,\mathbf{k} \quad ,$$

$$C : x = t^2 \quad , \quad y = t^4 \quad , \quad z = -t^9 \quad , \quad 0 \le t \le 1 \quad .$$

Ans:

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12. Evaluate the line integral

$$\int_C 5y\,dx + 5x\,dy + 6z\,dz \quad ,$$

where $C : x = t^2\,,\ y = t\,,\ z = t^2\,,\ 0 \leq t \leq 1.$

Ans.:

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13. Evaluate

$$\int\int\int_E \frac{1}{x^2+y^2+z^2}\, dV \quad ,$$

where E is the portion of the ball $x^2+y^2+z^2 \leq 4$ that lies in the octant

$$\{(x,y,z)|x \geq 0\,,\; y \geq 0\,,\; z \geq 0\} \quad .$$

Ans.:

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14. Evaluate the triple integral

$$\int\int\int_E 10yz\,dV \quad ,$$

where

$$E = \{(x,y,z) \,|\, 0 \le z \le 1\,,\; 0 \le y \le z\,,\; y+z \le x \le y+2z\,\} \quad .$$

Ans.:

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15. Find the Jacobian of the transformation from (u, v, w)-space to (x, y, z)-space.

$$x = 2u + 3v + w^2 \quad , \quad y = -u + 2v \quad , \quad z = v + 3w,$$

at the point $(u, v, w) = (1, 1, 0)$.

Ans.:

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16. Evaluate the integral

$$\int\int_D \frac{8e^8}{\pi} e^{-2x^2-2y^2}\, dA \quad ,$$

where D is the region

$$\{(x,y)\,|\,x^2+y^2 \le 4, x \ge 0, y \le 0\}$$

Ans.:

17.. Calculate the iterated integral

$$\int_0^1 \int_0^1 \int_0^1 (2x + 2y + 2z)\, dx\, dy\, dz \quad .$$

Ans.:

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18. Find the maximum value of the function

$$f(x, y) = 3x + 4y \quad ,$$

subject to the constraint $x^2 + y^2 = 25$.

Ans.:

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19. Find the local maximum and minimum **values** and saddle point(s) of the function $f(x,y) = x^3 + y^3 - 3xy$

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local maximum value(s):

local minimum value(s):

saddle point(s):

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20. Sketch the region of integration and change the order of integration.

$$\int_0^1 \int_0^x F(x,y)\,dy\,dx + \int_1^2 \int_0^{2-x} F(x,y)\,dy\,dx$$

Ans.:
