Dr. Z's Math251 Handout #0

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Problem Type 12.4b: (a) Find a vector orthogonal to the plane through the points $P = (p_1, p_2, p_3)$, $Q = (q_1, q_2, q_3)$,$R = (r_1, r_2, r_3)$. (b) Find the area of the triangle PQR.

Example Problem 12.4b: (a) Find a vector orthogonal to the plane through the points $P = (2, 1, 5)$, $Q = (-1, 3, 4)$,$R = (3, 0, 6)$. (b) Find the area of the triangle PQR.

Steps

1. Compute the vector **PQ** by subtracting Q from P and the vector **PR** by subtracting R from P.

2. Take the cross product of **PQ** and **PR**.

Example

1.

$$\mathbf{PQ} = (-1,3,4) - (2,1,5) = \langle -3, 2, -1 \rangle$$

$$\mathbf{PR} = (3,0,6) - (2,1,5) = \langle 1, -1, 1 \rangle$$

$$\mathbf{PQ} \times \mathbf{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 2 & -1 \\ 1 & -1 & 1 \end{vmatrix} \quad .$$

$$= \mathbf{i} \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} - \mathbf{j} \begin{vmatrix} -3 & -1 \\ 1 & 1 \end{vmatrix} + \mathbf{k} \begin{vmatrix} -3 & 2 \\ 1 & -1 \end{vmatrix}$$

$$= \mathbf{i} + 2\mathbf{j} + \mathbf{k} = \langle 1, 2, 1 \rangle$$

Ans. to First Part: The vector $\langle 1, 2, 1 \rangle$ is orthogonal to the plane of the triangle PQR.

3. Take the magnitude of the vector you found in step 2. This is the **area of the parralelogram**. To get the **area of the parralelogram**, divide it by 2.

3.

$$|\langle 1, 2, 1\rangle| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6} \quad .$$

So the area of the parallelogram formed by PQ and PR is $\sqrt{6}$ and the area of triangle PQR is half of that, $\sqrt{6}/2$.

Ans. to Second Part: The area of triangle PQR is $\sqrt{6}/2$.